Math 2802
Applications of Linear Algebra

School of Mathematics
Georgia Institute of Technology

Resources There are links from Canvas.

Everything is on the course webpage.

Calendar

How to succeed in class

You'll also find:

- Course administration: the names of your TAs, their office hours, your recitation location, etc.
- Course organization: details about grading, homework and exams, etc.
- Calendar: what will happen on which day, links to daily slides, quizzes, practice exams, solutions, etc.

Resources There are links from Canvas.

MyMathLab: Homework assignments and link to Learning Catalytics.

Socrative app: from time to time.

Piazza: this is where to ask questions, and where I'll post announcements.

- Click on Piazza Tab to join the master group MATH 2802-N1-N3.
- Better to use the Piazza app

Piazza/Learning Catalytics polls conducted in lecture will be used for your participation grade, so bring a computer or smartphone to class with you.

Grading Scheme – see syllabus for details

- *Comprehensive final exam:* 30%
- *3 Midterm exams:* 15% each
- *Weekly Quizzes:* 15% (lowest grade dropped)
- *Homework and participation:* 5% each (2-3 lowest grades dropped)

- **optional** *Written assignment:* due April 19th. (Tentative due to class size)

There is no quiz nor exams makeups.

Applications of Linear Algebra

Motivation and Overview

Linear. Algebra.

What was Linear Algebra?

Linear

- having to do with lines/planes/etc.
- For example, $x + y + 3z = 7$, not $\sin, \log, x^2$, etc.

Algebra

- **solving equations** involving numbers and symbols
- from al-jebr (Arabic), meaning reunion of broken parts
- 9^{th} century Abu Ja'far Muhammad ibn Muso al-Khwarizmi

But these are the easiest kind of equations! I learned how to solve them in 7th grade!

Applications of Linear Algebra

Chemistry: Balancing reaction equations

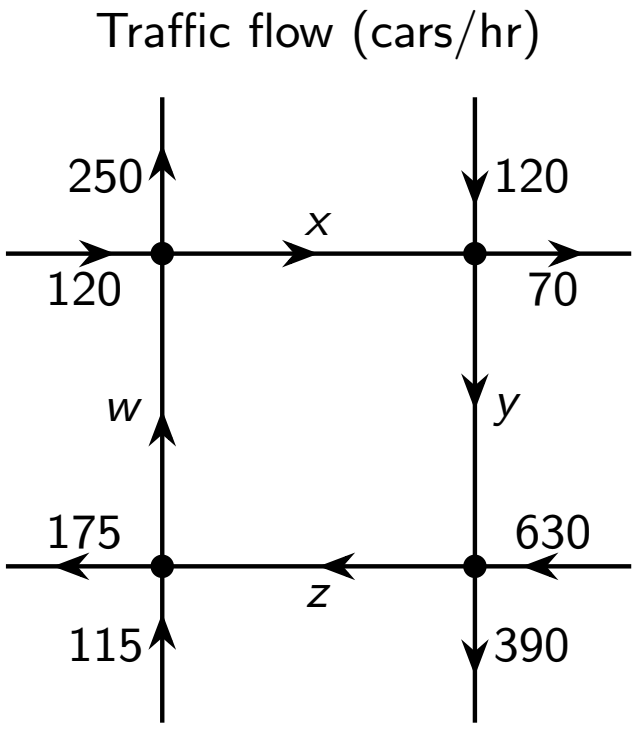
$$\underline{\ x\ }\ C_2H_6 + \underline{\ y\ }\ O_2 \to \underline{\ z\ }\ CO_2 + \underline{\ w\ }\ H_2O$$

$\rightsquigarrow$ system of linear equations, one equation for each element.

$$\begin{aligned} 2x &= z \\ 6x &= 2w \\ 2y &= 2z \end{aligned}$$

Applications of Linear Algebra

Civil Engineering: How much traffic flows through the four labeled segments?



⇝ system of linear equations:

$$\begin{aligned} w + 120 &= x + 250 \\ x + 120 &= y + 70 \\ y + 630 &= z + 390 \\ z + 115 &= w + 175 \end{aligned}$$

Linear Algebra in Engineering

Large classes of engineering problems, no matter how huge, can be reduced to linear algebra:

$$Ax = b \quad \text{or}$$
$$Ax = \lambda x$$

"... and now it's just linear algebra"

Overview of MATH 1553

Solve the matrix equation $Ax = b$

- Solve systems of linear equations using matrices, row reduction, etc.
- Solve systems of linear equations with varying parameters using parametric forms for solutions, the geometry of linear transformations, the characterizations of invertible matrices, and determinants.

Solve the matrix equation $Ax = \lambda x$

- Solve eigenvalue problems through the use of the characteristic polynomial.
- Understand the dynamics of a linear transformation via the computation of eigenvalues, eigenvectors, and diagonalization.

Almost solve the equation $Ax = b$

- Find best-fit solutions to systems of linear equations that have no actual solution using least squares approximations.

Why a whole course (again)?

Engineers need to solve *lots* of equations in *lots* of variables.

$$\begin{aligned}
3x_1 + 4x_2 + 10x_3 + 19x_4 - 2x_5 - 3x_6 &= 141 \\
7x_1 + 2x_2 - 13x_3 - 7x_4 + 21x_5 + 8x_6 &= 2567 \\
-x_1 + 9x_2 + \tfrac{3}{2}x_3 + x_4 + 14x_5 + 27x_6 &= 26 \\
\tfrac{1}{2}x_1 + 4x_2 + 10x_3 + 11x_4 + 2x_5 + x_6 &= -15
\end{aligned}$$

Often, it's enough to know some information about the set of solutions without having to solve the equations at all!

Also, what if one of the coefficients of the x_i is itself a parameter— like an unknown real number t?

In real life, the difficult part is often in recognizing that a problem can be solved using linear algebra in the first place:

need **conceptual** understanding.

What to Expect in MATH 2802

Your previous MATH 1553 probably focused on how to do computations.

- Row reduce an augmented matrix to solve a system of equations
- Compute the determinant of a matrix.
- Compute the rank of a matrix.
- Find the eigenvalues and eigenvectors of a matrix.

This is important, **but** Wolfram Alpha can do all these problems better than any of us can. Nobody is going to hire you to do something a computer can do better.

> If a computer can do the problem better than you can, then it's just an algorithm: this is not real problem solving.

So what are we going to do?

- About half the material focuses on reviewing MATH 1553 topics; mainly linear algebra computations—that is still important.
- The other half is on *conceptual* understanding of linear algebra and its *applications*. This is much more subtle: it's about figuring out *what question* to ask the computer, or whether you actually need to do any computations at all.

Applications of Linear Algebra

Geometry and Astronomy: Find the equation of a circle passing through 3 given points, say $(1,0)$, $(0,1)$, and $(1,1)$. The general form of a circle is $a(x^2+y^2)+bx+cy+d=0$.

$\rightsquigarrow$ system of linear equations:

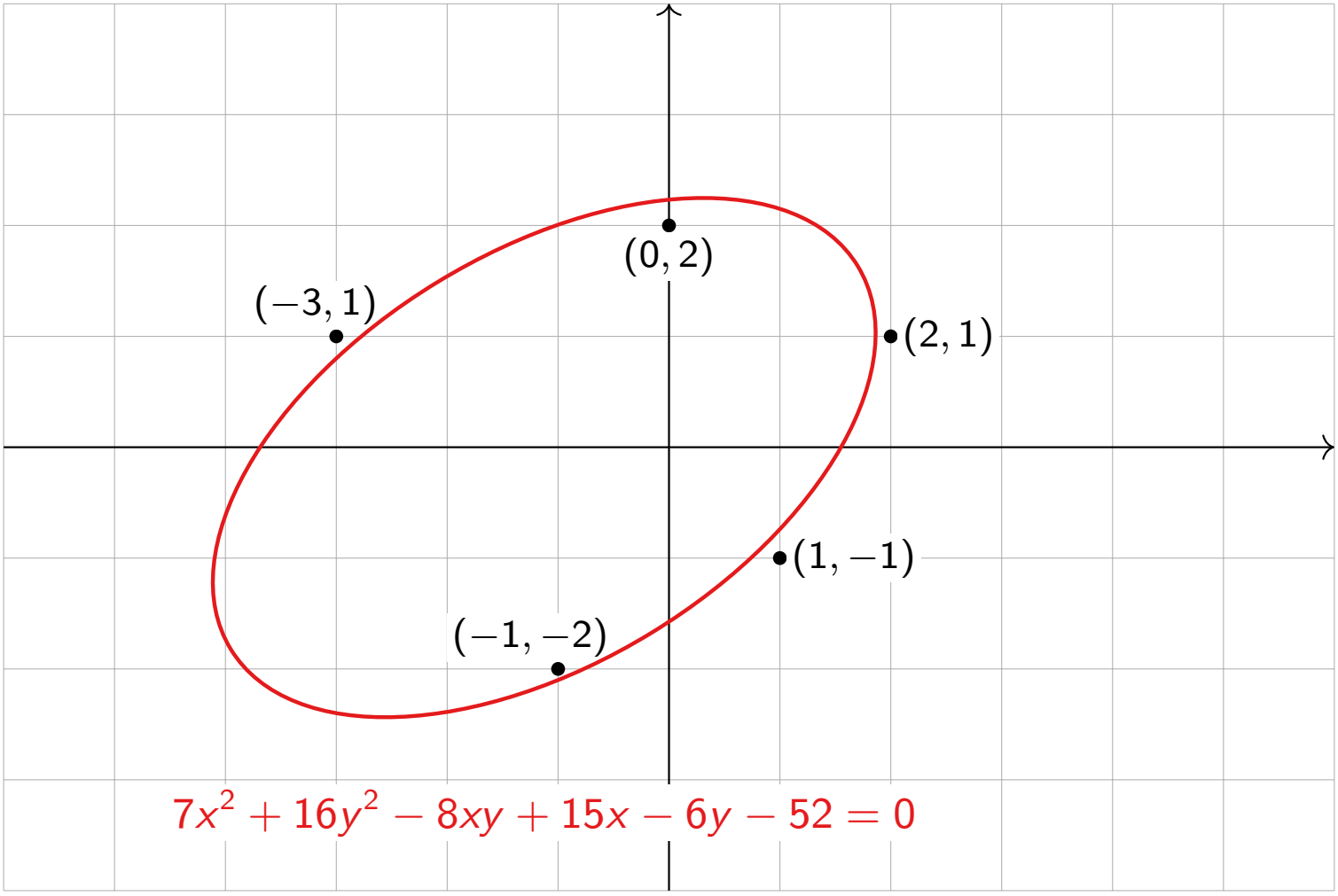
$$\begin{aligned} a+b\quad\quad+d&=0\\ a\quad\quad+c+d&=0\\ 2a+b+c+d&=0 \end{aligned}$$

Very similar to: compute the orbit of a planet:

$$ax^2+by^2+cxy+dx+ey+f=0$$

Data modeling: best fit ellipse

Picture



Remark: Gauss invented the method of least squares to do exactly this: he predicted the (elliptical) orbit of the asteroid Ceres as it passed behind the sun in 1801.

Applications of Linear Algebra

Biology: In a population of rabbits. . .

- half of the new born rabbits survive their first year
- of those, half survive their second year
- the maximum life span is three years
- rabbits produce 0, 6, 8 rabbits in their first, second, and third years

If I know the population in 2016 (in terms of the number of first, second, and third year rabbits), then what is the population in 2017?

$\rightsquigarrow$ system of linear equations:

$$\begin{aligned} 6y_{2016} + 8z_{2016} &= x_{2017} \\ \tfrac{1}{2}x_{2016} &= y_{2017} \\ \tfrac{1}{2}y_{2016} &= z_{2017} \end{aligned}$$

Question

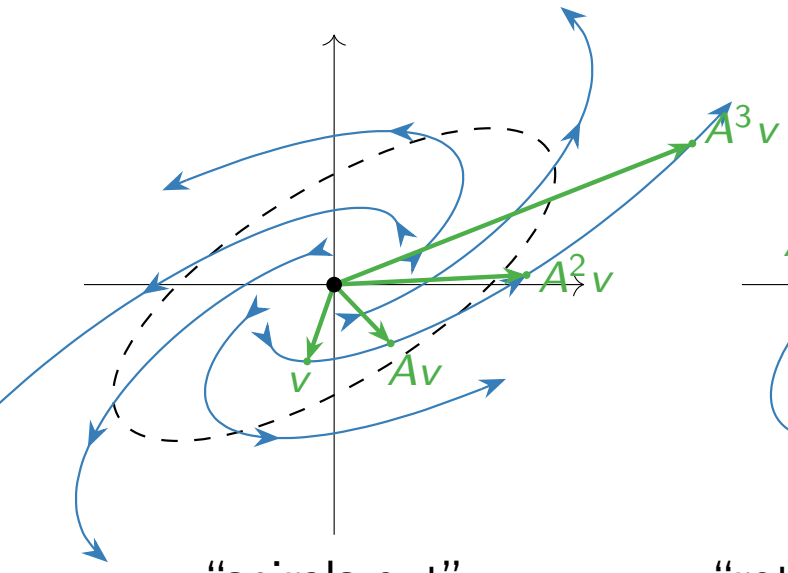
Does the rabbit population have an asymptotic behavior? Is this even a linear algebra question? Yes, it is!

Classification of 2×2 Matrices with no Real Eigenvalue

Triptych

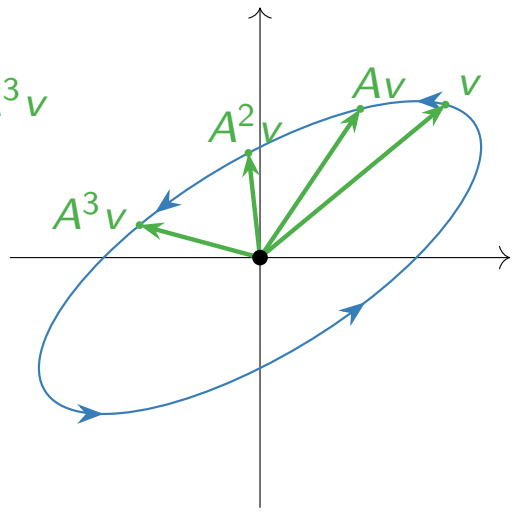
Pictures of *sequence of vectors* $v, Av, A^2v, \ldots$, a real matrix with not real eigenvalues, depending on the length of eigenvalues.

$|\lambda| > 1$



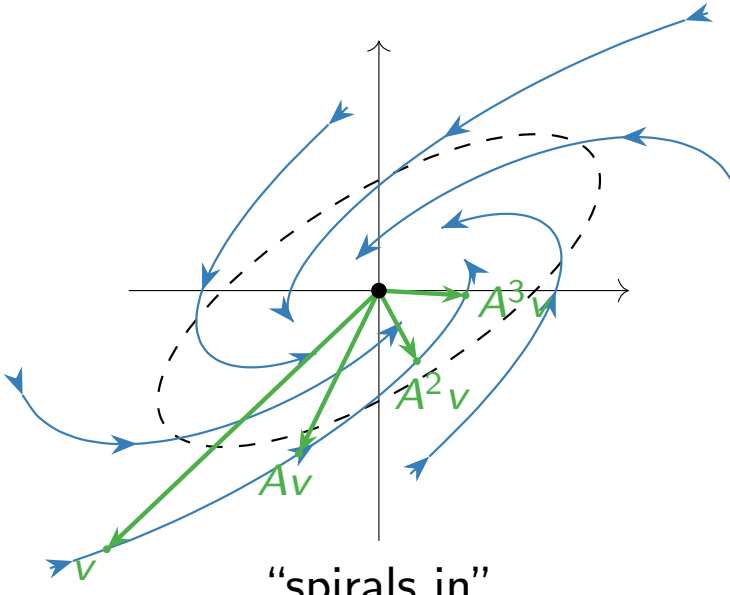
"spirals out"

$|\lambda| = 1$



"rotates around an ellipse"

$|\lambda| < 1$



"spirals in"

A great application of Linear Algebra

Google: "The 25 billion dollar eigenvector." Each web page has some importance, which it shares via outgoing links to other pages

$\rightsquigarrow$ system of linear equations (in gazillions of variables).

Larry Page flies around in a private 747 because he paid attention in his linear algebra class!

Stay tuned!