Chapter 1

Linear Equations

Introduction

What is $\mathbb{R}^n$?

Line, Plane, Space, . . .

Recall that $\mathbf{R}$ denotes the collection of all real numbers, i.e. the number line. It contains numbers like $0, -1, \pi, \frac{3}{2}, \ldots$

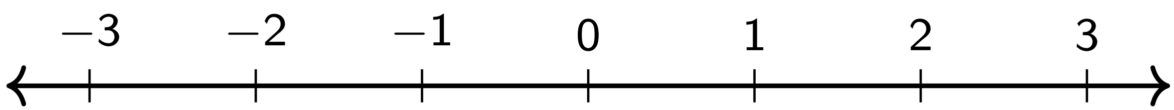
Definition

Let n be a positive whole number. We define

$$\mathbf{R}^n = \text{all ordered } n\text{-tuples of real numbers } (x_1, x_2, x_3, \ldots, x_n).$$

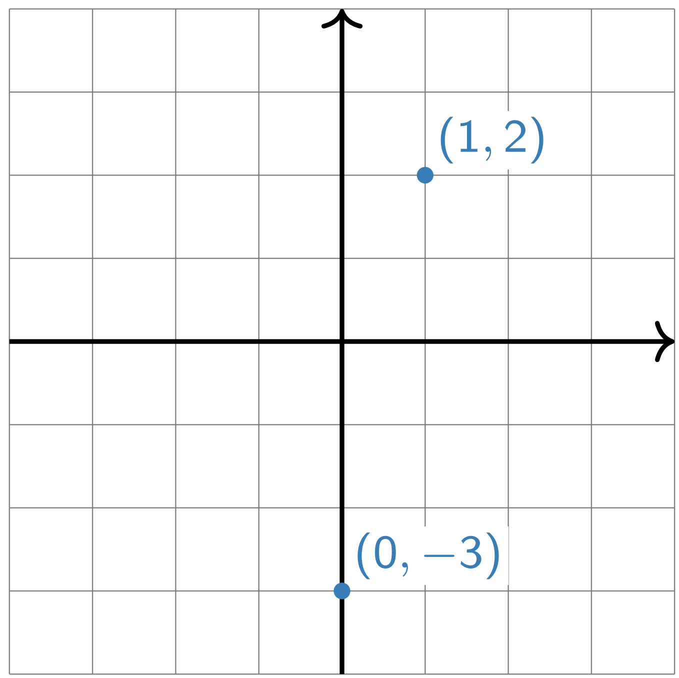
Example

When $n = 1$, we just get $\mathbf{R}$ back: $\mathbf{R}^1 = \mathbf{R}$. Geometrically, this is the *number line*.



Example

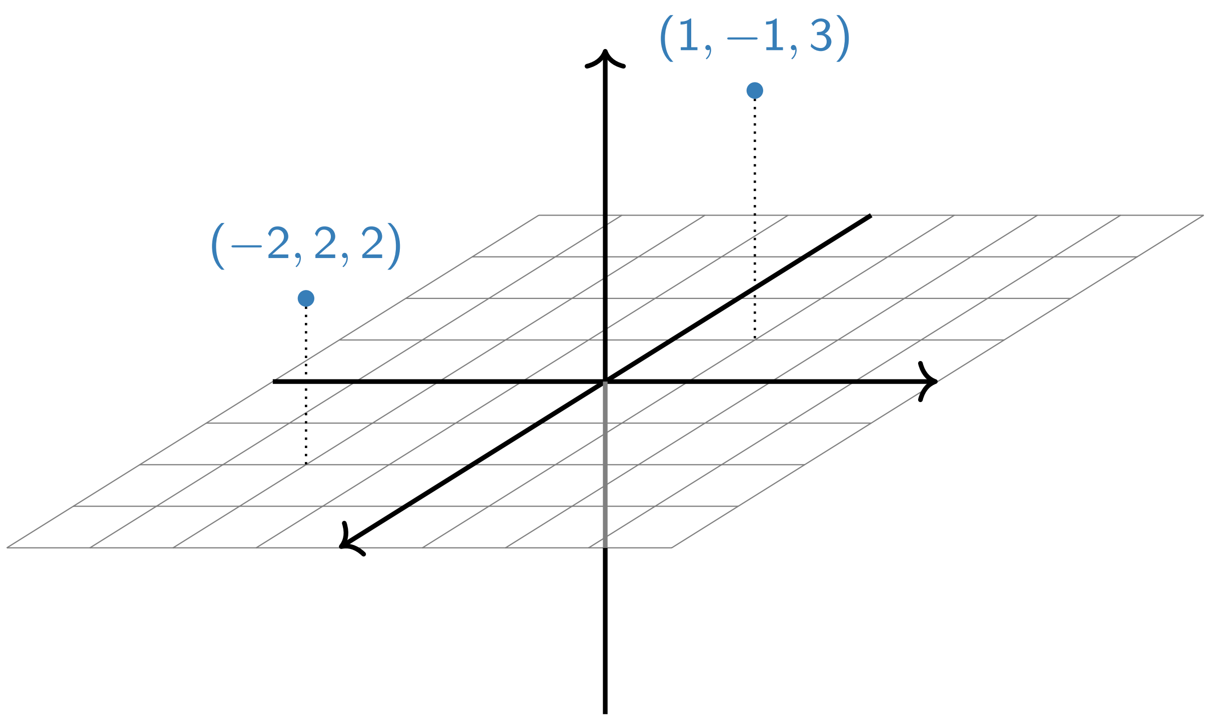
When $n = 2$, we can think of $\mathbf{R}^2$ as the *plane*. This is because every point on the plane can be represented by an ordered pair of real numbers, namely, its x- and y-coordinates.



We can use the elements of $\mathbf{R}^2$ to *label* points on the plane, but $\mathbf{R}^2$ is not defined to be the plane!

Example

When $n = 3$, we can think of $\mathbf{R}^3$ as the *space* we (appear to) live in. This is because every point in space can be represented by an ordered triple of real numbers, namely, its x-, y-, and z-coordinates.



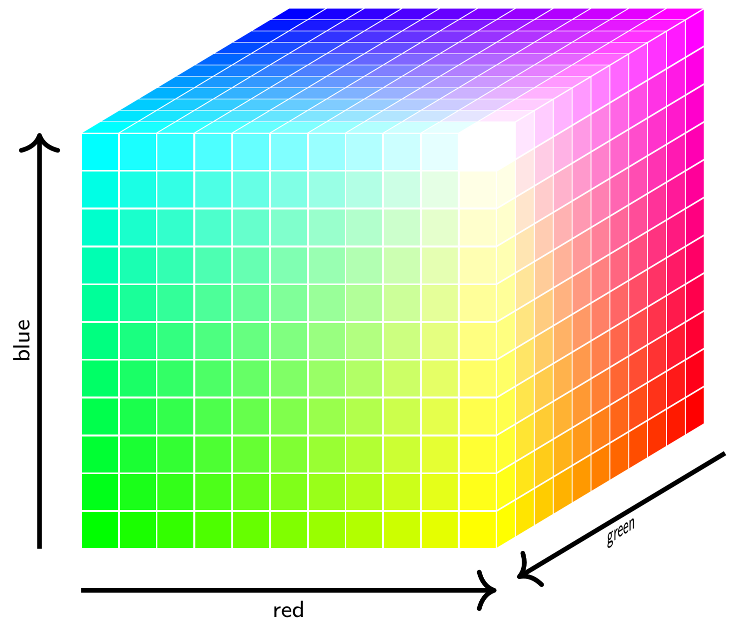
Again, we can use the elements of $\mathbf{R}^3$ to *label* points in space, but $\mathbf{R}^3$ is not defined to be space!

Line, Plane, Space, . . .
Continued

Example

All colors you can see can be described by three quantities: the amount of red, green, and blue light in that color. So we could also think of $\mathbf{R}^3$ as the space of all *colors*:

$$\mathbf{R}^3 = \text{all colors } (r, g, b).$$



Again, we can use the elements of $\mathbf{R}^3$ to *label* the colors, but $\mathbf{R}^3$ is not defined to be the space of all colors!

Line, Plane, Space, . . .

Continued

So what is $\mathbf{R}^4$? or $\mathbf{R}^5$? or $\mathbf{R}^n$?

. . . go back to the *definition*: ordered n-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They're still "geometric" spaces, in the sense that our intuition for $\mathbf{R}^2$ and $\mathbf{R}^3$ sometimes extends to $\mathbf{R}^n$, but they're harder to visualize.

$\rightsquigarrow$ Last time we could have used $\mathbf{R}^4$ to label the number of molecules involved in the combustion reaction.

$$\underline{\ x\ }\ C_2H_6 + \underline{\ y\ }\ O_2 \to \underline{\ z\ }\ CO_2 + \underline{\ w\ }\ H_2O$$

We'll make definitions and state theorems that apply to any $\mathbf{R}^n$, but we'll only draw pictures for $\mathbf{R}^2$ and $\mathbf{R}^3$.