Georgia Tech Center for Academic Success

MATH 1554 READING DAY STUDY SESSION WORKSHEET

Problems

1. A 5×4 matrix $A = [\vec{a_1} \quad \vec{a_2} \quad \vec{a_3} \quad \vec{a_4}]$ has all non-zero columns, and $\vec{a_4} = 2\vec{a_1} + 3\vec{a_2} + 5\vec{a_3}$. Find a non-trivial solution to $A\vec{x} = \vec{0}$.
2. For what values of h, if any, are the columns of A linearly dependent? $A = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & 1 \\ h & 1 & 0 \end{bmatrix}$
3. For what values of h is $\vec{b}$ in the plane spanned by $\vec{a_1}$ and $\vec{a_2}$?
$$\vec{a_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{a_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -1 \\ 1 \\ h \end{bmatrix}$$
4. Express the solution to $A\vec{x} = \vec{0}$ in parametric vector form, where $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 0 & 1 & 2 \end{bmatrix}$
5. Write down the standard matrix A of $T : \mathbb{R}^2 \to \mathbb{R}^2$ with $T(\vec{x}) = -\vec{x}$.
6. Find the domain and codomain of the linear transformation T given by the standard matrix
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 7 & 3 \\ 2 & 5 & -1 \end{bmatrix}$$
Is this linear transformation one-to-one? Is it onto?
7. Let $A = \begin{bmatrix} -5 & 2 \\ -1 & -3 \end{bmatrix}$. Find its eigenvalue(s) and find an invertible matrix P and a (rotation-scaling) matrix C such that $A = PCP^{-1}$.
8. W is the set of all vectors of the form $\begin{bmatrix} x \\ x+y \\ y \end{bmatrix}$. With of the following vectors are in $W^{\perp}$?
$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$
9. Identify all values of a, b, and c, if any, so that the columns of U are mutually orthogonal.
$$U = \begin{bmatrix} 3 & 2 & 2 \\ -4 & 1 & b \\ 2 & a & c \end{bmatrix}$$
10. Use the Gram-Schmidt process to construct an orthonormal basis of the column space of A.
11. Let $A = QR$, where $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ -2 & -2 \end{bmatrix}$, $Q = \dfrac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -2 & 2 \end{bmatrix}$ Compute the upper triangular matrix R.
12. Give an example of a 2×2 matrix that is in echelon form, is orthogonally diagonalizable, but is not invertible.

13. True or False?

(i) If the set of vectors $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent, so is every pair of vectors $\{\vec{u}, \vec{v}\}$, $\{\vec{u}, \vec{w}\}$, and $\{\vec{v}, \vec{w}\}$.

(ii) If every pair of vectors $\{\vec{u}, \vec{v}\}$, $\{\vec{u}, \vec{w}\}$, and $\{\vec{v}, \vec{w}\}$ is linearly independent, so is the set of vectors $\{\vec{u}, \vec{v}, \vec{w}\}$.

(iii) For any two vectors $\vec{u}$ and $\vec{v}$, we have $Span\{\vec{u}, \vec{v}\} = Span\{\vec{u}, 2\vec{u} + 3\vec{v}, 4\vec{v}\}$.

(iv) If $\vec{u}$ and $\vec{v}$ are two distinct nonzero vectors, then there are exactly to vectors in $Span\{\vec{u}, \vec{v}\}$.

(v) The transformation given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 x_2 \\ x_2 \end{bmatrix}$ is linear.

(vi) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a projection onto the x_1-axis. The range of T is $\mathbb{R}^2$.

(vii) The transformation given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 1 \\ x_2 \end{bmatrix}$ is linear.

(viii) A linear map $T : \mathbb{R}^2 \to \mathbb{R}^3$ can be onto.

(ix) The composition $S \circ T$ of two one-to-one linear maps is one-to-one.

(x) The range of a one-to-one linear map $T : \mathbb{R}^2 \to \mathbb{R}^3$ may be a line.

(xi) The eigenvalues of a square matrix A are the same as the eigenvalues of its reduced row echelon form.

(xii) If $\vec{u}$ and $\vec{v}$ are eigenvectors corresponding to the same eigenvalue λ, then every linear combination of $a\vec{u} + b\vec{v}$ with $a, b \in \mathbb{R}$ (except the zero vector) is an eigenvector.

(xiii) The geometric multiplicity of an eigenvalue is less than or equal to the algebraic multiplicity.

(xiv) All upper triangular 3×3 stochastic matrices are not regular.

(xv) If A is a diagonalizable matrix, then $\lambda = 0$ is not an eigenvalue of A.

(xvi) An $n \times n$ matrix with n distinct eigenvalues is diagonalizable.

(xvii) If complex λ is an eigenvalue, then so is $-\lambda$.

(xviii) Every real 3×3 matrix must have a real eigenvalue.

(xix) For any three vectors $\vec{x}$, $\vec{y}$, and $\vec{z}$ we have $(\vec{x} \cdot \vec{y})\vec{z} = (\vec{y} \cdot \vec{z})\vec{x}$.

(xx) Let $\vec{x} \cdot \vec{y} > 0$. Then the angle between $\vec{x}$ and $\vec{y}$ is less than $90°$.

(xxi) Every orthogonal set of nonzero vectors $\{\vec{x}, \vec{y}, \vec{z}\}$ is linearly independent.

(xxii) Let $\hat{y}$ be the orthogonal projection of a vector $\vec{y}$ onto the subspace $W \subset \mathbb{R}^n$. Then the transformation $T(\vec{y}) = \hat{y}$ is linear.

(xxiii) The inverse of an orthogonal matrix Q equals Q^T.

(xxiv) A least square solution $\hat{x}$ to $A\vec{x} = \vec{b}$ always satisfies $A\vec{x} = \vec{b}$.

(xxv) A least square solution $\hat{x}$ to $A\vec{x} = \vec{b}$ minimizes the distance $||A\vec{x} - \vec{b}||$. That is, the distance is the shortest for $\vec{x} = \hat{x}$.

(xxvi) A $n \times n$ symmetric matrix A will always have n real and distinct eigenvalues.

(xxvii) A $n \times n$ symmetric matrix A will have algebraic multiplicity = geometric multiplicity for each of its eigenvalues.

(xxviii) If a matrix A is orthogonally diagonalizable, then A^k is also orthogonally diagonalizable for all $k \in \mathbb{Z}^+$.

(xxix) The eigenvalues of $A^T A$ are always real for any $m \times n$ matrix A.

(xxx) A negative definite matrix cannot be invertible.

(xxxi) Matrices A and A^T have the same non-zero singular values.

(xxxii) For any matrix A, $A^t A$ has non-negative, real eigenvalues.