MATH 2802
MIDTERM EXAMINATION 2

Name		Section	

Please **read all instructions** carefully before beginning.

- There are 6 problems in the exam and the maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- You may use the last page as scratch paper
- Good luck!

1. [2 points each] Circle **T** if the statement is always true and circle **F** if it is ever false. The matrices here are $n \times n$.

a) **T** **F** If the geometric multiplicities of eigenvalues in A sum up to n, then A is diagonalizable.

b) **T** **F** A vector $v = (v_1, \ldots, v_n)$ is steady-state vector of a stochastic matrix A if $Av = v$ and the length $|v| = \sqrt{v_1^2 + \ldots + v_n^2} = 1$.

c) **T** **F** If P is invertible, then $\det(PDP^{-1}) = \det(D)$.

d) **T** **F** Then dimension of the column space of A is called $rank(A)$.

e) **T** **F** The determinant of an invertible matrix is always positive.

f) **T** **F** A and $B = 2A$ are $n \times n$ matrices. If $\det(A) = 4$, then $\det(B) = 8$.

2. [10 points]
Consider the decomposition of $A = PDP^{-1}$ with

$$P = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 0 \\ 0 & 1/3 \end{pmatrix}$$

a) Draw both the 1-eigenspace and the 1/3-eigenspace of A.

b) Provide an eigenvector of A with eigenvalue 1.

c) Evaluate $A^{100} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

d) Write a formula for D^n.

3. [8 points] In this problem, show your work and justify your answers.

a) Is 5 an eigenvalue of $A = \begin{pmatrix} -2 & 7 \\ -5 & 10 \end{pmatrix}$?

b) Is $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ an eigenvector of $B = \begin{pmatrix} 7 & 6 & -1 \\ 0 & 4 & 8 \\ 3 & -8 & 17 \end{pmatrix}$?

c) Find the 3-eigenspace of $C = \begin{pmatrix} 4 & -7 & 0 & 2 \\ 0 & 3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

d) Is C diagonalizable?

4. [10 points] Consider a matrix A which is row-equivalent to:

$$A = \begin{pmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 8 & 0 & 5 \\ 0 & 2 & 5 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

a) Find a basis for $Col(A)$

b) What is the dimension for $Nul(A)$?

c) Justify your answer in b) *(There are several possible correct answers)*

d) If the dimension of V is m then:

- Any m linearly independent vectors in V form ____________ for V .
- Any m vectors that ____________ form ____________ for V .

5. [5pts]

There is a car rental with locations at the airport, Midtown and Marietta. After a comprehensive study the administration knows that:

- A car rented at the airport has 20% chance of being returned to the Midtown location and 10% chance of being returned to the Marietta location.
- A car rented at the Midtown location has 10% chance of being returned to the airport and 10% chance of being returned to the Marietta location.
- A car rented at the Marietta location has 30% chance of being returned to the airport and 30% chance of being returned to the Midtown location.

a) Find the transition matrix Q (make transitions from airport, midtown and Marietta correspond to columns 1,2 and 3 respectively).

b) If the steady-state vector of Q is $w = \frac{1}{28}\begin{pmatrix} 9 \\ 15 \\ 4 \end{pmatrix}$. What percentage of time will a car in the rental be returned to the Midtown location?

6. [7 points] An economy of coal and electric sectors has production matrix $C = \begin{pmatrix} 0 & .5 \\ .6 & .2 \end{pmatrix}$ and a demand of $d = \begin{pmatrix} 50 \\ 30 \end{pmatrix}$ is requested. Use Leontief's inverse matrix to determine the production level x necessary to satisfy the demand d.

Hint: remember that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Scoring Table

Please do not write on this area.

1	2	3	4	5	6	Total

[Scratch work below this line]