MATH 2802
MIDTERM EXAMINATION 3

Name		**Section**	

Please **read all instructions** carefully before beginning.

- There are 6 problems in the exam and the maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- You may use the last page as scratch paper
- Good luck!

1. [2 points each] Circle **T** if the statement is always true and circle **F** if it is ever false. The matrices here are $n \times n$.

a) **T** **F** If z is orthogonal to both v and w, then z is in the orthogonal complement of $Span\{v, w\}$.

b) **T** **F** The orthogonal projection of y onto W depends on the basis selected for W.

c) **T** **F** If $\{v_1, v_2, v_3\}$ is an orthogonal basis of W then $\{v_1, v_2, 5v_3\}$ is an orthogonal basis for W.

d) **T** **F** If P is orthonormal then $P^{-1} = P^T$

e) **T** **F** The quadratic form Q is positive definite if for all non-zero vectors $Q(x) \geq 0$.

2. [6 points] Let $W = Span\{v_1, v_2\}$ with $v_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Find $y = proj_W \begin{pmatrix} 1 \\ 1 \\ 7 \end{pmatrix}$. Justify your answer.

3. [6 points] Let $v_1 = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$, $v_2 = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ and $v_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.

a) Compute $v_1 \cdot v_2$, $v_2 \cdot v_3$ and $v_1 \cdot v_3$ (Show your work).

b) Is $\{v_1, v_2, v_3\}$ a basis of $\mathbf{R}^3$? Justify your answer.

4. [8pts]

Find the best approximation to $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ by vectors of the form $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. What is the error in your approximation?

5. [10 points]

Apply the Gram-Schmidt procedure to the vectors $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ and $v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ and give an orthonormal basis for $Span\{v_1, v_2, v_3\}$.

6. [10 points] Let $Q(x_1, x_2) = 3x_1^2 + 2x_1x_2 + 3x_2^2$. There is a matrix P such that the change of variables $y = P^{-1}x$ gives the following form $Q(y) = ay_1^2 + by_2^2$.

a) Find the matrix associated to $Q(x) = x^T Ax$

b) Find an orthogonal decomposition for $A = PDP^T$

c) Give the values of a and b

d) Is the quadratic form $Q(x)$ indefinite? Justify your answer.

Scoring Table

Please do not write on this area.

1	2	3	4	5	6	Total

[Scratch work below this line]