Name: **Recitation Section:**

Math 2802 N1-N3 Quiz

Solutions

The quiz has a total of 10 points and you have 15 minutes. Read carefully and clearly justify how you obtained your answers.

1. [3 points] Find all the values of h for which the following vectors are linearly independent:

$$v_1 = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} \quad v_2 = \begin{pmatrix} 6 \\ -10 \\ 20 \end{pmatrix} \quad v_3 = \begin{pmatrix} -2 \\ 10 \\ h \end{pmatrix}$$

Solution.

Place the vectors as columns of a matrix and row reduce to find the number of pivot columns:

$$\begin{pmatrix} 1 & 6 & -2 \\ -2 & -10 & 10 \\ 4 & 20 & h \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & -2 \\ 0 & 2 & 6 \\ 0 & -4 & h+8 \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & -2 \\ 0 & 2 & 6 \\ 0 & 0 & h+20 \end{pmatrix}$$

The vectors are linearly independent if there is one pivot in each column. Thus the vectors are linearly independent whenever $h \neq 20$.

2. [3 points] Is the following transformation $T : \mathbf{R}^n \to \mathbf{R}^n$ invertible? Justify your answer.

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6x - 2y + z \\ -10x + 10y - 2z \\ 20x + 4z \end{pmatrix}$$

Solution.

The matrix corresponding to T is given by $A = \begin{pmatrix} 6 & -2 & 1 \\ -10 & 10 & -2 \\ 20 & 0 & -4 \end{pmatrix}$.

Note that the column vectors of A are linearly **independent** (by the problem above). Thus the transformation is invertible (by the invertible matrix theorem).

Turn the page!

3. [4 points]
For this problem, use the following row-reduction of A.

$$A = \begin{pmatrix} 1 & -3 & 4 & -1 & 5 \\ 3 & -9 & 7 & -2 & 9 \\ -2 & 6 & -3 & 1 & -4 \\ -1 & 3 & 6 & -1 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & 4 & -1 & 5 \\ 0 & 0 & -5 & 1 & -6 \\ 0 & 0 & 5 & -1 & 6 \\ 0 & 0 & 10 & -2 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & 4 & -1 & 5 \\ 0 & 0 & -5 & 1 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Let $A = LU$ where L is a lower triangular matrix and U is an echelon form.

a) The dimensions of L: ________ rows × ________ columns

b) The dimensions of U: ________ rows × ________ columns

c) Construct the matrices L and U.

Solution.

a) The dimensions of L: 4×4

b) The dimensions of U: 4×5

c) The matrix U is the (not-reduced) echelon form obtained in the row reduction above.

$$U = \begin{pmatrix} 1 & -3 & 4 & -1 & 5 \\ 0 & 0 & -5 & 1 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

To obtain L, the first pivot was located on the first column and the entries from the pivot and below were $\begin{pmatrix} 1 \\ 3 \\ -2 \\ -1 \end{pmatrix}$. The second pivot was located on the third column and the entries from the pivot and below were $\begin{pmatrix} 5 \\ -1 \\ 10 \end{pmatrix}$; normalizing so that the first entry is a one: $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

The are no more pivots. Now, start with the matrix $L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix}$ and replace the vectors we have in the first two columns. The remaining undeclared entry on the third columns should then be zero.

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -2 & -1 & 1 & 0 \\ -1 & -2 & 0 & 1 \end{pmatrix}$$