Name: **Recitation Section:**

Math 2802 N1-N3 Quiz

Solutions

The quiz has a total of 10 points and you have 15 minutes. Read carefully and clearly justify how you obtained your answers.

1. [6 points]

Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be defined as $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+3y-z \\ 4x+6y-2z \end{pmatrix}$

a) Find the standard matrix for T.

b) Draw a picture of the range T.

c) Is it onto? If not, find a vector b in $\mathbf{R}^2$ which is not in the range. (It is enough to use the picture in (b).)

Solution.

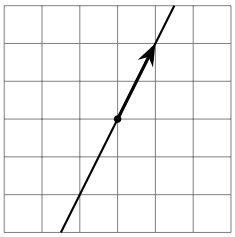
a) **[3 pts]** We have to plug in the unit coordinate vectors to get the columns:

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}.$$

Therefore the standard matrix for T is

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 6 & -2 \end{pmatrix}.$$

b) **[2 pts]** The range of T is the span of the columns of the standard matrix. All three columns lie on the line spanned by $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, so the range is just this line.



c) **[1 pts]** The range of T is a line in $\mathbf{R}^2$, so it is strictly smaller than the codomain. Hence T is not onto. Looking at the picture, we see that, for instance, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is not in the range.

2. [4 points] In a certian region, about 6% of a city's population moves to the surrounding suburbs each year, and about 4% of the suburban population movees

into the city. In 2017, there where 10,000,000 residents in the city and 800,000 in the suburbs.

Set up a difference equation that describes this situation, where x_0 is the initial population in 2017.

Solution.

[3 pts] Let $A = \begin{pmatrix} .94 & .4 \\ .6 & .96 \end{pmatrix}$, this matrix represents the flow between city and suburban population. The first column represents the flow from the city; from a 100% percent population of the city (that is represented by e_1) we have

$$Ae_1 = \begin{pmatrix} .94 \\ .4 \end{pmatrix}$$

94% stays in the city and 4% moves to the suburbs.

Similarly, the second column represents the flow from the suburbs; from a 100% percent population of the suburbs (that is represented by e_2) we have

$$Ae_2 = \begin{pmatrix} .4 \\ .96 \end{pmatrix}$$

4% moves to the city and 96% stays in the suburbs.

[1 pt] If we are given the population on 2017, we can set $x_0 = \begin{pmatrix} 10 \\ 000 \\ 000 \\ 80 \\ 000 \end{pmatrix}$ and the population on 2018 will be given by $x_1 = Ax_0$.