Name: **Recitation Section:**

Math 2802 N1-N3 Quiz

Solutions

The quiz has a total of 10 points and you have 15 minutes. Read carefully and clearly justify how you obtained your answers.

1. [2 points] *(Hint: use a specific example if you want to test your answer)*
Let A and B be a 3×3 matrix with $\det(A) = 8$. If the rows of A and B are place as follows: $A = \begin{pmatrix} - & r_1 & - \\ - & r_2 & - \\ - & r_3 & - \end{pmatrix}$ $B = \begin{pmatrix} - & r_3 & - \\ - & r_2 & - \\ - & r_1 & - \end{pmatrix}$.

Then $\det(B)$ equals: ________________________

Solution.

To go from A to B we have to perform 3 row swaps for example

$$\begin{pmatrix} - & r_1 & - \\ - & r_2 & - \\ - & r_3 & - \end{pmatrix} \sim \begin{pmatrix} - & r_2 & - \\ - & r_1 & - \\ - & r_3 & - \end{pmatrix} \sim \begin{pmatrix} - & r_2 & - \\ - & r_3 & - \\ - & r_1 & - \end{pmatrix} \sim \begin{pmatrix} - & r_3 & - \\ - & r_2 & - \\ - & r_1 & - \end{pmatrix}$$

Therefore, we have $\det(B) = (-1)^3 \det(A) = -8$.

2. [1 point] Let B be a 3×3 matrix If $rank(B) = 2$, then
Dimension of $Nul(B)$ equals ________________________

Solution.

By the rank theorem $dim(NulB) + \text{rank}(B) = 3$. So $dim(NulB) = 1$

3. Start with this row reduction of A to find... $A = \begin{pmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{pmatrix} \sim$

$\begin{pmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

a) [2 points] the value for $rank(A)$

b) [2 points] a basis for $ColA$,

c) [3 points] a basis for $NulA$,

Solution.

a) The value for $rank(A) = 2$ because there are two pivots in A

b) The column pivots are the first and the third ones. So a basis for $ColA$ is

$$\left\{ \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} \right\}$$

c) For a basis of $NulA$, we find the solution to the homogeneous equation of $Ax = 0$:

$$\begin{aligned} x_1 - 3x_2 \ + \ 6x_3 \ + \ 9x_4 &= 0 \\ +4x_3 \quad +5x_4 &= 0 \end{aligned}$$

Solving for x_1 and x_3 (leaving x_2 and x_4 free) gives

$$\begin{aligned} x_1 \quad &= 3x_2 - \tfrac{3}{2}x_4 \\ + \ x_3 &= \quad - \ \tfrac{5}{4}x_4 \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} \frac{-3}{2} \\ 0 \\ \frac{-5}{4} \\ 1 \end{pmatrix}$$

$$= x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \frac{x_4}{4} \begin{pmatrix} -6 \\ 0 \\ -5 \\ 4 \end{pmatrix}$$

So basis for $NulA$ are, for example,

$$\left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{-3}{2} \\ 0 \\ \frac{-5}{4} \\ 1 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -6 \\ 0 \\ -5 \\ 4 \end{pmatrix} \right\}$$