Name: _____   **Recitation Section:** _____

Math 2802 N1-N3 Quiz
Solutions

The quiz has a total of 10 points and you have 15 minutes. Read carefully and clearly justify how you obtained your answers.

1. [3 points]

  a) If $\lambda + 5$ is a factor of the characteristic polynomial $\det(A - \lambda I)$,
  then an eigenvalue of A is: _____

  b) Give an example of a 2×2 matrix that is diagonalizable but not invertible.

Solution.

  a) Eigenvalue is -5, since this is the value of λ that makes $\det(A - \lambda I) = 0$.

  b) For example $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ is already diagonal but is not invertible.

2. [3pts] Let $A = \begin{pmatrix} 10 & -5 \\ 5 & 10 \end{pmatrix}$

  a) Compute the characteristic polynomial of A.

  b) How many real eigenvalues does A have?

Solution.

  a) $f(\lambda) = (10 - \lambda)^2 + 25$

  b) If $f(\lambda) = 0$ then $(10 - \lambda)^2 = -25$. If there was a real root to $f(\lambda)$ then that would mean that a squared real number (i.e $(10 - \lambda)^2$) is negative, which is impossible. Alternatively, from the quadratic formula we see that the roots of $f(\lambda)$ are only complex numbers.

3. [4pts] Let $A = \begin{pmatrix} 4 & -3 & 6 & -8 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & -2 \end{pmatrix}$.

  a) What is the algebraic multiplicity of eigenvalue 4?

  b) What is the maximum dimension of the 4-eigenspace?

  c) Describe the steps to find the 4-eigenspace of A.

Solution.

  a) 2; you can read this off from the diagonal entries.

  b) 2.

c) Find the solution set for the equation $(A - 4I)x = 0$ and this is precisely the 4-eigenspace of A.