Name: **Recitation Section:**

Math 2802 N1-N3 Quiz

Solutions

The quiz has a total of 10 points and you have 15 minutes. Read carefully and clearly justify how you obtained your answers.

1. [6 points] Let $A = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$. The least-square solution to $Ax = b$ is $\widehat{x} = \begin{pmatrix} 1/3 \\ -1/3 \end{pmatrix}$. Compute the error associated to this least-squares solution.

(*Hint: The error is the distance between two vectors*)

Solution.

The desired solution x should satisfy $Ax = b$. Instead, the best solution is $\widehat{x}$, which computes

$$A\widehat{x} = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1/3 \\ -1/3 \end{pmatrix} = \frac{1}{3}\left(\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}\right) = \frac{1}{3}\begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}.$$

Therefore, the error is

$$||Ax - A\widehat{x}|| = ||b - A\widehat{x}|| = ||\begin{pmatrix} -1/3 \\ 2/3 \\ -1/3 \end{pmatrix}|| = \frac{1}{3}\sqrt{1+4+1} = \sqrt{2/3}$$

2. [4 pts] Consider a best fit parabola $y = \beta_2 x^2 + \beta_1 x$ for the following data points. Provide a design matrix A and observation vector y so that the least-squares solution to $A\binom{\beta_1}{\beta_2} = y$ gives the paramaters β_1, β_2. (*Do not solve the least-squares problem*)

x	1	2	3	4	5
y	2.8	3.7	4.6	4.8	5.2

Solution.

The parabola $\beta_2 x^2 + \beta_1 x$ predicts that for, say $x = 3$, the value of the second coordinate is $3\beta_1 + 9\beta_2$, and the observation is 3.7. This gives a system of equations were the variables are β_1 and β_2. In matrix form:

$$A\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = y \implies \begin{pmatrix} 1 & 1 \\ 2 & 4 \\ 3 & 9 \\ 4 & 16 \\ 5 & 25 \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 2.8 \\ 3.7 \\ 4.6 \\ 4.8 \\ 5.2 \end{pmatrix}$$