Name: **Recitation Section:**

Math 2802 N1-N3 Quiz

Solutions

The quiz has a total of 10 points and you have 15 minutes. Read carefully and clearly justify how you obtained your answers.

1. [2pts] Let $Q(x_1, x_2, x_3) = 2x_1x_2 - 4x_1x_3 - x_2x_3 - x_3^2$. Give the matrix associated to the quadratic form $Q(x)$.

Solution.

The diagonal terms correspond to coefficient of squared variables, the coefficients of the cross-product $x_i x_j$ is evenly divided into the (i, j) and (j, i) entries:

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 1 & 0 & -1/2 \\ -2 & -1/2 & -1 \end{pmatrix}$$

2. [4 pts] Classify the following quadratic functions:

a) $Q(x_1, x_2) = 2x_2^2 - 2x_1^2$,

b) $Q(x_1, x_2) = -3x_1^2$.

Solution.

a) $Q(x_1, x_2) = 2x_2^2 - 2x_1^2$ is indefinite,

b) $Q(x_1, x_2) = -3x_1^2$ is negative semidefinite.

3. [4 pts] The 2×2 matrix A can be written as $A = PDP^{-1}$ with $D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$ and P orthonormal matrix. If u_1, u_2 are the columns vectors of P, write the formula for the spectral decomposition of A.

Solution.

$A = d_1 u_1 u_1^T + d_2 u_2 u_2^T$. Recall that $u_1 u_1^T$ and $u_2 u_2^T$ are 2×2 matrices.