Sections 1.6 & 1.10

Applications and linear Models

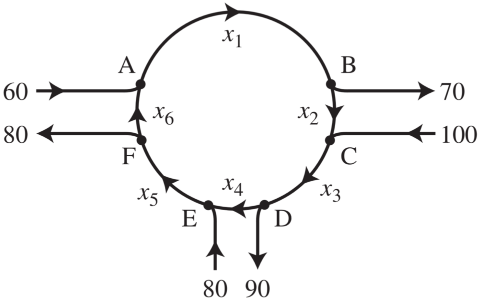
Linearity...

- *Reflects some property* of the system being modeled
- Easily adapted for computer *calculations*
- Approximate phenomena when parameters are held within some boundaries

  e.g. derivatives are used as *linear approximations*

Zero-sum systems: Traffic flows and Electrical Networks

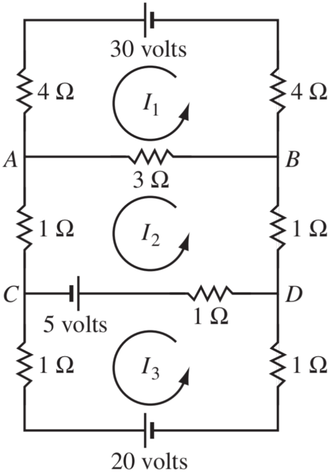
- Zero-sum property of *flows in/out* around every node.



- **Constraints:** If the flow directions are mandatory (as traffic lanes) then their *values must remain non-negative*.

Zero-sum systems: Electrical Networks

- Zero-sum property of *voltages drops/sources* around every loop.
- The resulting currents I_i for each loop satisfy the traffic flow properties!



$$I_1 \underset{\uparrow \atop \mathbf{r}_1}{\begin{bmatrix} 11 \\ -3 \\ 0 \end{bmatrix}} + I_2 \underset{\uparrow \atop \mathbf{r}_2}{\begin{bmatrix} -3 \\ 6 \\ -1 \end{bmatrix}} + I_3 \underset{\uparrow \atop \mathbf{r}_3}{\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}} = \underset{\uparrow \atop \mathbf{v}}{\begin{bmatrix} 30 \\ 5 \\ -25 \end{bmatrix}}$$

Linear combinations: Constructing a ~~Weight-Loss~~ Nutritious Diet

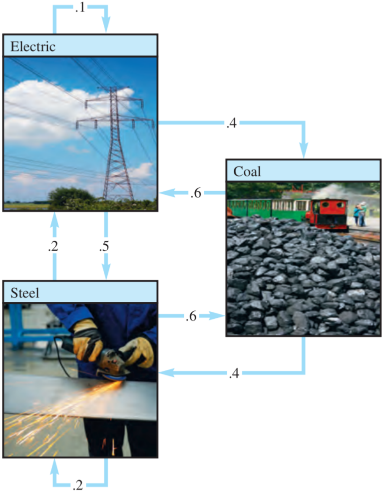
Or for any system of balanced/reliable supplies from different distributors.

Amounts (g) Supplied per 100 g of Ingredient				**Amounts (g) Supplied by Cambridge Diet in One Day**
Nutrient	**Nonfat milk**	**Soy flour**	**Whey**	
Protein	36	51	13	33
Carbohydrate	52	34	74	45
Fat	0	7	1.1	3

- *Constraints:* The vector of weights for each product/distributor has to be non-negative to make sense in its context.

Linear combinations: Economy equilibrium prices

- **Equilibrium prices** can be assigned to the total outputs of the various sectors in such a way that the income of each sector balances its expenses
- *Constraints:* The prices have to be positive to make sense in this context.



Distribution of Output from:			
Coal	**Electric**	**Steel**	**Purchased by:**
.0	.4	.6	Coal
.6	.1	.2	Electric
.4	.5	.2	Steel

Difference equations: Population migration

This is a **difference equation**: $Ax_n = x_{n+1}$

Annual percentage migration between city and suburbs.

From:

City	Suburbs	To:
.95	.03	City
.05	.97	Suburbs

$$\begin{bmatrix} .95 & .03 \\ .05 & .97 \end{bmatrix}$$

If you know *initial population* x_0, what happens *in 10 years* x_{10}?

Difference equations: Population growth

How to predict a population of rabbits with given **dynamics**:

1. half of the newborn rabbits *survive* their first year;
2. of those, half *survive* their second year;
3. their maximum *life span* is three years;
4. Each rabbit gets $0, 6, 8$ *baby rabbits* in their three years, respectively.

Approach: Each year, count the population **by age**:

$$v_n = \begin{pmatrix} f_n \\ s_n \\ t_n \end{pmatrix} \text{ where } \begin{cases} f_n & = \text{first-year rabbits in year } n \\ s_n & = \text{second-year rabbits in year } n \\ t_n & = \text{third-year rabbits in year } n \end{cases}$$

The *dynamics say*:

$$\overbrace{\begin{pmatrix} f_{n+1} \\ s_{n+1} \\ t_{n+1} \end{pmatrix}}^{v_{n+1}} = \begin{pmatrix} 6s_n + 8t_n \\ f_n/2 \\ s_n/2 \end{pmatrix} = \overbrace{\begin{pmatrix} 0 & 6 & 8 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} f_n \\ s_n \\ t_n \end{pmatrix}}^{Av_n}.$$

Plug in a computer:

v_0	v_{10}	v_{11}
$\begin{pmatrix}1\\2\\3\end{pmatrix}$	$\begin{pmatrix}9459\\2434\\577\end{pmatrix}$	$\begin{pmatrix}19222\\4729\\1217\end{pmatrix}$
$\begin{pmatrix}3\\7\\9\end{pmatrix}$	$\begin{pmatrix}30189\\7761\\1844\end{pmatrix}$	$\begin{pmatrix}61316\\15095\\3881\end{pmatrix}$
$\begin{pmatrix}16\\4\\1\end{pmatrix}$	$\begin{pmatrix}16384\\4096\\1024\end{pmatrix}$	$\begin{pmatrix}32768\\8192\\2048\end{pmatrix}$

Notice any patterns?

1. Each segment of the population *essentially doubles* every year: $Av_{11} \approx 2v_{10}$.
2. The ratios get close to $(16 : 4 : 1)$:

$$v_{11} \approx (\text{big}\#) \cdot \begin{pmatrix}16\\4\\1\end{pmatrix}.$$

New terms coming (after midterm 1): eigenvalue, and eigenvector