Math 2802 N1-N3 Worksheet 10

April 6th, 2018

1. Let $Q(x_1,x_2,x_3) = x_1^2 - x_3^2 + 3x_1x_2 - 4x_1x_3 + x_2x_3$. Give the associated matrix to the quadratic form $Q(x)$.

2. Classify the following quadratic functions according to: positive/negative definite, positive/negative semidefinite or indefinite.

a) $Q(x_1,x_2,x_3) = -15x_3^2$,

b) $Q(x_1,x_2,x_3) = 2x_1^2 - 10x_3^2$,

c) $Q(x_1,x_2,x_3) = 2x_1^2 + 3x_2^2 + 10x_3^2$.

3. Let A be a 2×2 matrix with eigenvalue λ. If the information of A is that $A = \begin{pmatrix} a & b \\ * & * \end{pmatrix}$.

Then one eigenvector with eigenvalue is $\begin{pmatrix} b \\ \lambda - a \end{pmatrix}$. Why?

4. The graph of the equation $8x_1^2 + 6x_1x_2 = 8$ corresponds to an hyperbola. Find a change of variables that removes the cross-product term from the equation and give the formula of the hyperbola using the new variables.

5. Compute the spectral decomposition for $A = PDP^{-1}$ with

$$P = \begin{pmatrix} .6 & .8 \\ .8 & -.6 \end{pmatrix} \qquad D = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

6. Find the orthogonal diagonalization $A = PDP^{-1}$ of A below (which has distinct eignevalues $-4, 4, 7$). Write down explicitly both P and P^{-1}.

$$A = \begin{pmatrix} 1 & 1 & 5 \\ 1 & 5 & 1 \\ 5 & 1 & 1 \end{pmatrix}$$

7. Consider $A = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

a) Find two orthogonal eigenvectors of A with eigenvalue 5.

b) Find an orthogonal diagonalization of A (*Hint:* v is eigenvector of A)