MATH 2802 N1-N3, WORKSHEET 3

JANUARY 26TH, 2018

Choose the correct answers below.

(1) Every linear transformation from $\mathbb{R}^n \to \mathbb{R}^m$ is a matrix transformation.

   (a) **True.** There exists a unique matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^n$.

   (b) **True.** There exists a unique matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^m$.

   (c) **True.** Every matrix transformation spans $\mathbb{R}^m$.

(2) A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps onto a unique vector in $\mathbb{R}^m$.

   (a) **False.** A mapping T is said to be one-to-one if each $b \in \mathbb{R}^m$ is the image of at most one $x \in \mathbb{R}^n$.

   (b) **False.** A mapping T is said to be one-to-one if each $b \in \mathbb{R}^m$ is the image of at least one $x \in \mathbb{R}^n$.

   (c) **True.** A mapping T is said to be one-to-one if each $b \in \mathbb{R}^m$ is the image of exactly one $x \in \mathbb{R}^n$.

   (d) **True.** A mapping T is said to be one-to-one if each $x \in \mathbb{R}^n$ has at least one image for $b \in \mathbb{R}^m$.

(3) The second little pig has decided to build his house out of sticks. The big bad wolf finds the pigs house and blows it down so that the house is rotated by an angle of $45°$ in a counterclockwise direction about the z-axis, and then projected onto the xy-plane. Find the matrix for this transformation.

Soultion.

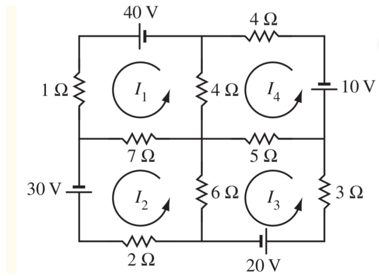
To compute the matrix for T, we have to compute $T(e_1), T(e_2)$ and $T(e_3)$.To see the picture, lets put ourselves above the xy-plane, looking downward. First the rotation is along the z-axis so the third coordinate does not change, followed by the projection which 'forget the third coordinate (which is zero for both e_1 and e_2), we have

$$T(e_1) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} \qquad T(e_2) = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$$

Rotating e_3 around the z-axis does nothing, and projecting onto the xy-plane sends it to zero, so $T(e_3) = 0$. Therefore, the matrix for T is

$$\begin{pmatrix}| & | & | \\ T(e_1) & T(e_2) & T(e_3) \\ | & | & |\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1 & 0\\1 & 1 & 0\\0 & 0 & 0\end{pmatrix}$$

(4) Exercise 7 from Section 1.10.



Soultion.
The system of linear equations is

$$\begin{aligned} 12I_1 - 7I_2 - 4I_4 &= 40 \\ -7I_1 + 15I_2 - 6I_3 &= 30 \\ -6I_2 + 14I_3 - 5I_4 &= 20 \\ -4I_1 - 5I_3 + 13 &= -10 \end{aligned}$$

Extra: The solution to the system is $\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} \approx \begin{pmatrix} 11.43 \\ 5.84 \\ 10.55 \\ 8.03 \end{pmatrix}$

(5) Exercise 12 from Section 1.10.

Soultion.
Let $A = \begin{pmatrix} .97 & .05 & .10 \\ .00 & .90 & .05 \\ .03 & .05 & .85 \end{pmatrix}$ be the matrix of distributions of cars returned to-and-from the three given locations.

The vector $x_0 = \begin{pmatrix} 295 \\ 55 \\ 150 \end{pmatrix}$ has entries corresponding to the number of cars at the airport, east and weast locations, respectively, on Monday. The vector corresponding to Tuesday's number of cars at each location is given by

$$x_1 = Ax_0.$$

Therefore, the **approximate** vector corresponding to Wednesday's number of cars at each location is

$$x_2 = Ax_1 = A(Ax_0) = A^2x_0.$$

Extra. Plugging in the values of A and x_0 gives

$$x_2 = A\begin{pmatrix} 303.9 \\ 57 \\ 139.1 \end{pmatrix} = \begin{pmatrix} 311.543 \\ 58.255 \\ 130.202 \end{pmatrix} \approx \begin{pmatrix} 312 \\ 58 \\ 130 \end{pmatrix}.$$