Math 2802 N1-N3 Worksheet 6

February 23rd, 2018

1. Determine whether the following statements are true or give a counterexample. Let A be $n \times n$ matrix

a) If $Ax = \lambda x$ then λ is an eigenvalue of A

b) If there are matrices P, D such that $A = PDP^{-1}$ then A is diagonalizable.

c) If A has n distinct eigenvalues then A is diagonalizable.

d) If A has only one eigenvalue with algebraic multiplicity n then A is not diagonalizable.

2. **Discuss:** What is the difference between algebraic multiplicity of an eigenvalue and geometric multiplicity of an eigenvalue.

3. Find the algebraic multiplicity and eigenspace of eigenvalue 5 for matrix

$$A = \begin{pmatrix} 5 & 5 & 0 & 2 \\ 0 & 2 & -3 & 6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

4. Let $A = PDP^{-1}$ with $P = \begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$. Draw the eigenspaces of 2 and $1/2$; and (approximately) draw x, Ax, A^2x, A^3x, A^4x; for $x = \begin{pmatrix} 0 \\ 10 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. If possible, do not compute powers of A.

5. Let $A = PDP^{-1}$ with $P = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. Compute A^k. Can you guess what are possible dynamics for $x, Ax, A^2x, \ldots$ depending on the values of a and b?