**Math 2802 N1-N3 Worksheet 8**

March 16th, 2018

**1.** Find an upper triangular matrix $R$ such that $A = QR$. Check your work.

$$A = \begin{pmatrix} -2 & 3 \\ 5 & 7 \\ 2 & -2 \\ 4 & 6 \end{pmatrix}, \qquad Q = \frac{1}{7}\begin{pmatrix} -2 & 5 \\ 5 & 2 \\ 2 & -4 \\ 2 & 4 \end{pmatrix}$$

**2.** Apply the Gram-Schmidt decomposition to the vectors

**a)** $v_1 = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

**b)** $v_1 = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 2 \\ -6 \\ 1 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

**3.** Decide whether the following sets form a basis for $\mathbf{R}^3$. If so, is it an orthogonal basis? If so, convert the orthogonal basis into an orthonormal basis.

**a)** $\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

**b)** $\left\{ \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} -3 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$

**c)** $\left\{ \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}, \begin{pmatrix} 1/2 \\ 0 \\ -1/2 \end{pmatrix}, \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \end{pmatrix} \right\}$

**4.** Find the distance from $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ to the plane in $\mathbf{R}^3$ spanned by $v_1 = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$.

**5.** If $y = \begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix}$, $u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $u_2 = \begin{pmatrix} -1 \\ 6 \\ -5 \end{pmatrix}$. Find a decomposition $y = \widehat{y} + z$, where $\widehat{y}$ is in $W = Span\{u_1, u_2\}$ and $z$ is in the orthogonal complement of $W$.