A pattern makes a conjecture

- $1 = 1$
- $4 = 1 + 3$
- $9 = 1 + 3 + 5$
- $16 = 1 + 3 + 5 + 7$
- $25 = 1 + 3 + 5 + 7 + 9$

- $1 = 2 \cdot 1 - 1$
- $3 = 2 \cdot 2 - 1$
- $5 = 2 \cdot 3 - 1$
- $7 = 2 \cdot 4 - 1$
- $9 = 2 \cdot 5 - 1$

The conjecture is a formula for squared positive integers:

$$n^2 \stackrel{?}{=} 1 + 3 + 5 + \cdots + (2n - 1).$$

The crux

The important idea (valid argument) in the proof is:

Given a propositional function $P(n)$

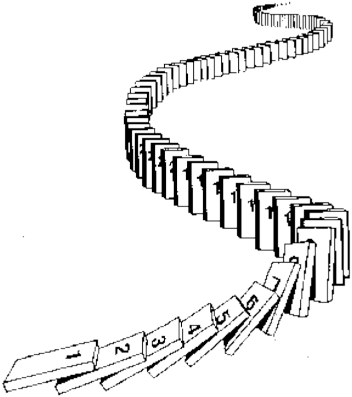
$$P(n) \to P(n+1)$$

is true for all positive integers n.

The starting point

We need to know for sure that $P(n)$ holds for some integer.

Well, in fact, we need to know
$P(1)$ is true!
because 1 is the first positive integer.



Induction principle

To prove $\forall n \in \mathbb{Z}^+ P(n)$ is true, complete two steps:

BASIS STEP:

Verify that the proposition $P(1)$ is true.

INDUCTIVE STEP:

Show that the conditional statement

$$P(k) \to P(k+1)$$

is true for all k positive integer.

The well ordering property

The proof from class makes one assumption.

Every nonempty set
of nonnegative integers
has a least element.

Mathematicians take this 'evident property' for granted; that is: it is an axiom.

Strong induction principle

To prove $\forall n \in \mathbb{Z}^+ P(n)$ is true, complete two steps:

BASIS STEP:

Verify that the proposition $P(1)$ is true.

INDUCTIVE STEP:

Show that the conditional statement

$$[P(1) \wedge P(2) \wedge \cdots \wedge P(k)] \rightarrow P(k+1)$$

is true for all k positive integer.

Pigeonhole principle

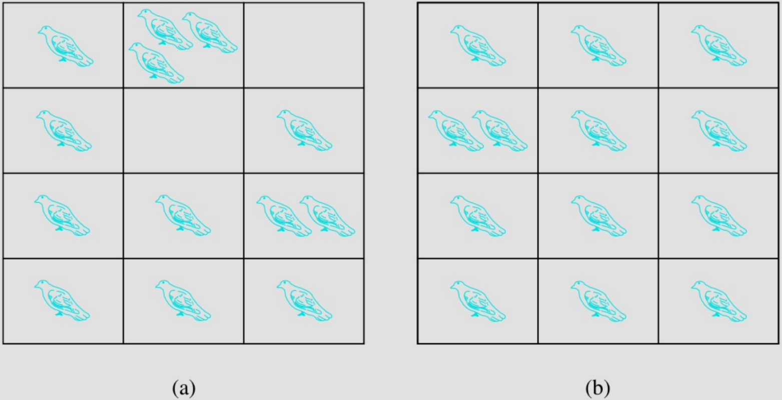


FIGURE 1 **There Are More Pigeons Than Pigeonholes.**

22 students and 7 different languages
(each student checked one language only).

Arabic	Chinese	Dutch	Farsi	Spanish	Urdu	Wolof
=	=			=	-	-
=	=			=		
=	-			=		
=						
-						
9	5	0	0	6	1	1

Then there is at least 4 students that speaks the same language.

Two versions

Simple version

If k is a positive integer and $k + 1$ or more objects are placed into k boxes, then there is at least one box containing at least two of the objects.

General version

If k, m are positive integers and $km + 1$ or more objects are placed into k boxes, then there is at least one box containing at least $m + 1$ of the objects.