Discrete Mathematics
MATH 363

Instructor Laura Eslava

McGill University

Winter 2016

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Instructor's whereabouts

Instructor: Laura Eslava

Office: Burnside Hall 1017

Office Hours: Wednesdays 14:30-16:00 hrs. or by appointment.

Do not hesitate to ask for help.

Prerequisites

- MATH 263 and MATH 264
- Restricted to students of the Faculty of Engineering

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- Interest for puzzles and riddles is a plus.

You will enjoy the course better if you see it as a challenge.

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- Late assignments will not be accepted.

Final and Midterm exams

Exams will count, either

Midterm 20% + Final 60%
or Final 80%

- The exams are closed-book and closed-notes.
- No calculators are permitted during the exams.

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Important date: Midterm on the 25th of February.

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- Mere copying is not permitted. Students must write up their own assignments separately.

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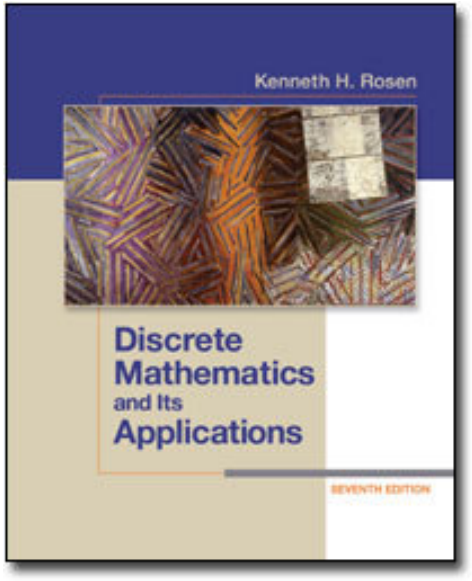
Kenneth H. Rosen, *Discrete Mathematics and its applications*, seventh edition, McGrawHill.

Webpage: Use the online learning center

www.mhhe.com/rosen



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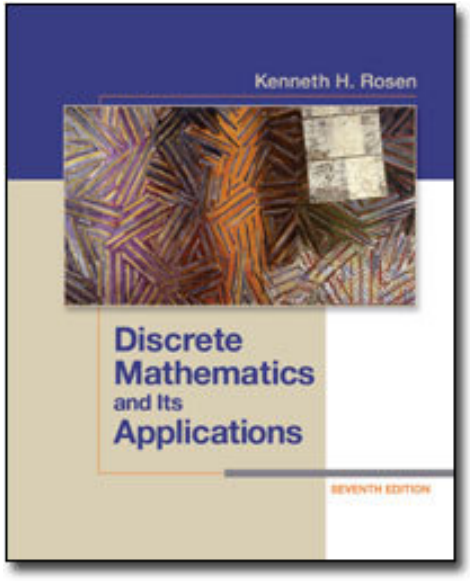

You know it: Mastery of the material requires that the students devote a significant amount of time to reading the textbook and solving problems.

Course webpage

There you will find:

- A log of the topics covered in each lecture.
- Assignments and solutions.
- Possibly extra material.

http://www.math.mcgill.ca/eslava/Courses/math363-w16

A list of riddles

In the following slides there will be several puzzles.

- Take a piece of paper and write down each of them, at the end of the class there will be time for you to think about them.

Topic 1: Propositional logic

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- Why is the following argument incorrect?

  If you do every problem in the textbook, then you will learn discrete mathematics.
  You learned discrete mathematics. Therefore, you did every problem in the textbook.

Topic 1: Propositional logic

Logic Puzzle

There is an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie.
You encounter two people A and B.
A: 'B is a knight'
B: 'The two of us are opposite types'
What are A and B?

Topic 2: Proofs and induction

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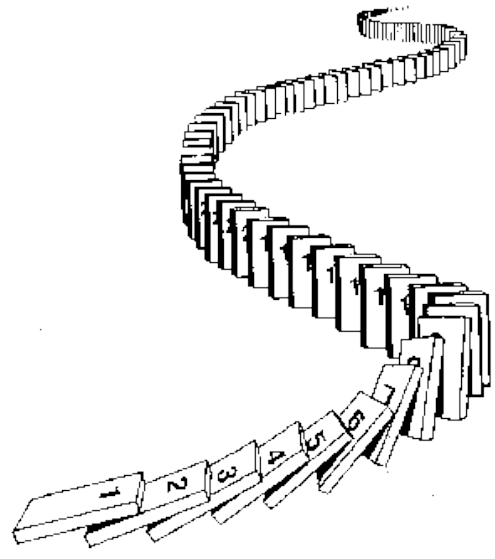
- Induction is a beautiful idea:

Tilt an infinite number of dominos with just one movement.

We will see what this means for mathematics.



Topic 3: Sets and functions

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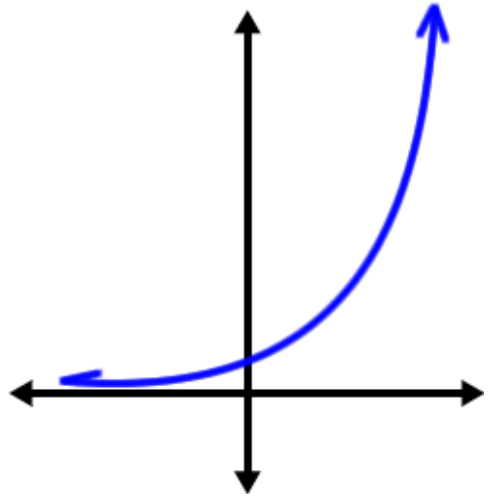
You will understand the difference between exponential, polynomial and linear growth.

- Danger of epidemics
- Why nuclear weapons are so destructive?
- How fast can computers become?



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- The requirement of grains on the 21th square demanded over a million grains of rice.

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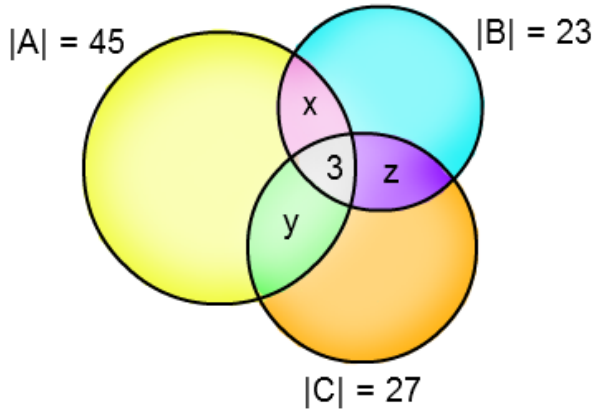
Count people in a subgroup

In a group of 89 people there are

45 Activists,
23 Biologists,
27 Canadians.

How many Canadian activists are there?



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- To encript messages and decode them.

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Measuring puzzle

You have a 3 and a 5 litre water container, each container has no markings except for that which gives you it's total volume. You also have a running tap. You must use the containers and the tap in such away as to exactly measure out 1 litres of water.

How would you measure it?

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- **Probabilistic algorithms** can be used to solve many problems that cannot be easily or practically solved by deterministic algorithms.

With a probabilistic algorithm, instead of always following the same steps when given the same input, as a deterministic algorithm does, the computer makes one or more random choices.

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Count how many students in the class share their birthdays?

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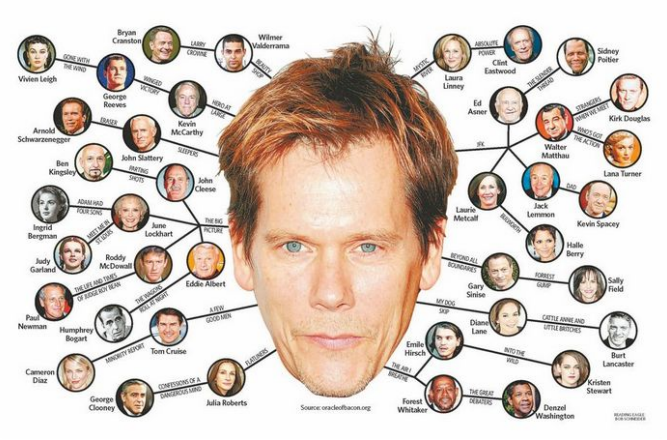
- Efficiently connecting cities with highways.



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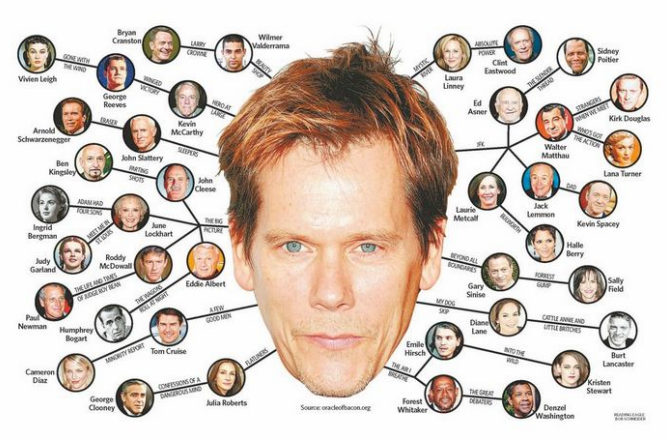

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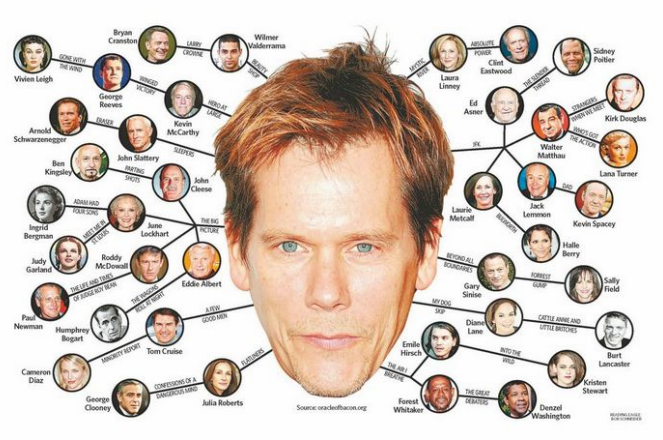
- Efficiently connecting cities with highways.
- Understanding interrelations between departments of a company.
- Representing collaboration networks.



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This will not be graded.

A poll

Write down in a paper:

1. Your name,
2. Major and year,
3. Why you are taking MATH 363,
4. Two of your favorite courses so far,
5. Two courses you are looking forward to take.

Hand this in to the instructor.

Have a nice beginning of term!