

Enumeration exercises

- Students and classes (easy)
 - There is a makeup midterm for two students at room 280 McDonald Eng. Building. There are 64 places to sit. In how many ways can the students sit down and take the exam? (they can sit next to each other)
 64×63
 - There were 85 students taking the midterm. As they hand in the exam the prof. would put the booklets in a pile. How many possible orderings of the booklets are there?
 $85!$
 - There are 24 students in a math contest. How many possible outcomes are there for the first, second and third prizes are there? (No ties are possible)
 $24 \times 23 \times 22$
- Poker questions
 - How distinct many pairs are there in a deck of cards?
 $13 \times C(4, 2) = 13 \times 6$
 - How distinct many 'three of a kind' are there in a deck of cards?
 $13 \times C(4, 3) = 13 \times 4$
 - How many distinct 'full-house' are there in a deck of cards?
 $13 \times C(4, 2) \times 12 \times C(4, 3) = 13 \times 6 \times 12 \times 4$
 - How many 'straight flush' are there in a deck of cards?
 4×10
- Car plates
 - If car plates consists of 3 letters and 3 digits. What is the number of distinct possible car plates?
 $26^3 \times 10^3$
 - If car plates consists of 3 distinct letters and 3 distinct digits. What is the number of distinct possible car plates?
 $(26 \times 25 \times 24) \times (10 \times 9 \times 8)$
 - If car plates consists of 3 letters and 3 digits. And there are no plates with 3 equal digits nor 3 equal letters. What is the number of distinct possible car plates?
 $(26^3 - 26) \times (10^3 - 10)$
- Passwords and bit-strings
 - The passwords on a computer system are 6 to 8 bits long and are formed by digits or uppercase letters. How many distinct passwords are there?
 $36^6 + 36^7 + 36^8$

- The passwords on a computer system are 6 bits long, are formed by digits or uppercase letters and at least one digit. How many distinct passwords are there?

$$36^6 - 26^6 = 10 \sum_{i=1}^6 26^{i-1} 36^{6-i}$$

- How many bit strings of length 8 either start with a 1 bit or end with the two bits 00?

$$2^7 + 2^6 - 2^5 = 2^5(4 + 1) = 2^5(4 + 2 - 1)$$

- Functions and sets

- Let A and B be finite sets of cardinality a and b respectively. How many functions from A to B are there?

$$b^a$$

- Let A and B be finite sets of cardinality a and b respectively. If $b \geq a$, how many injective functions from A to B are there?

$$P(b, a) = b \times (b - 1) \times \dots \times (b - a + 1)$$

- Let A and B be finite sets of cardinality a and b respectively. If $a > b$, how many one-to-one functions from A to B are there?

$$0$$

- If a set A has cardinality k. What is the cardinality of the power set of A?

$$2^k$$

- Students and classes (hard)

- There are 100 students in a math contest. There will be 10 gold medals 15 silver medals and 30 bronze medals. In how many ways can these prizes be delivered?

$$C(100, 10) \times C(90, 15) \times C(75, 30) = C(100, 55) \times C(55, 10) \times C(45, 15)$$

- There is a makeup midterm for two students at room 280 McDonald Eng. Building. The student will sit in the first row, where there are 8 places, and can not sit together. In how many ways can the students sit down and take the exam?

$$2 \times (6 + 5 + 4 + 3 + 2 + 1) = (2 \times 5) + (5 \times 6) = 42$$

- There are 24 students in a class of robotics. They will split the work in 4 teams of 6; each team in charge of a distinct task. In how many ways can the teams be formed?

$$C(24, 6) \times C(18, 6) \times C(12, 6) = \frac{24!}{(6!)^4}$$

- There are 24 students will make projects, in teams of 6 people, for a class. In how many ways can the teams be formed?

$$[C(24, 6) \times C(18, 6) \times C(12, 6)]/4! = \frac{24!}{4!(6!)^4}$$