

MATH 363 Discrete Mathematics

Midterm

- Write down your solutions, **no justification is needed** (+1pt).

1. Simplify: $\neg[(p \vee q) \vee \neg q] \wedge p$.
2. Draw the Venn Diagram of the following set $(B \cup C) \setminus A$.
3. List the elements of $S = \{n \in \mathbb{N} : n < 52, 7 \text{ divides } n\}$.
4. Determine if the following is true
 - $n!$ is $O(n^n)$,
 - $n^5 - n$ is $O(n^4)$.
5. Define when a compound proposition is a Tautology.
6. State the pigeonhole principle.
7. Give the encoding function of Caesar's cipher.
8. Describe how to encode x using the RSA encryption with key $(n = 3 \cdot 11, e = 7)$.

- Write down your solutions and **show your work** (+2pt).

1. Negate: $\forall x \in \mathbb{R} \forall y \in \mathbb{R} (xy \neq 0, \text{ or } y = 0 \text{ or } x = 0)$.
2. Let $U = \{1, 3, 5, 7, 9, 11\}$, $A = \{n \in U : 3 \leq n \leq 9\}$ and $B = \{n \in U : n \geq 6\}$.
Use a bit string to describe the set $A \cap \bar{B}$.

3. Let $f : \mathbf{Z} \rightarrow \mathbf{R}$ be defined by $f(n) = n^2$.

– What is the domain of f ?

– Is f injective?

4. Give the prime decomposition of $12!$.

5. Solve the congruence $4x = 9 \pmod{15}$.

6. Compute the following sum $\sum_{i=1}^4 |4 - 2i|$.

• Write down your **solutions** (+2pt) and **justify** (+1pt).

1. Give a big-O estimate for $(72n^2 - 50n)(3n + \log n^3)$.

2. Prove that if n is an integer, then 2 divides $n^2 + n$.

3. Let $0 < d < n$ be integers.

How many positive integers not exceeding n are divisible by d ?