

MATH 363 Discrete Mathematics

Midterm

- Write down your solutions, **no justification is needed**. (+1pt)

1. Simplify: $\neg[\neg(p \vee q) \wedge \neg p]$.
2. Draw the Venn Diagram of the following set $C \setminus (A \cup B)$.
3. List the elements of $S = \{n \in \mathbb{N} : n < 35, n = 3 \pmod{7}\}$.
4. Determine if the following is true.
 - $n^2 \log n$ is $O(n^3)$,
 - $n!$ is $O(n^2)$.
5. Define when a compound proposition is a Contradiction.
6. State the two steps of a proof by induction.
7. Give the decoding function of Caesar's cipher.
8. Describe how to encode x using the RSA encryption with key $(n = 5 \cdot 7, e = 5)$.

- Write down your solutions and **show your work** (+2pt).

1. Negate: $\forall x \in \mathbb{N} \exists y \in \mathbb{N} (x \text{ divides } y \text{ and } y^2 \text{ divides } x)$.
2. Let $U = \{0, 2, 4, 6, 8, 10\}$, $A = \{n \in U : 3 \leq n \leq 9\}$ and $B = \{n \in U : n \leq 5\}$.
Use a bit string to describe the set $\bar{A} \cup B$

3. Let $f : \mathbf{Z} \rightarrow \mathbf{R}$ be defined by $f(n) = n^2$.

– What is the codomain of f ?

– What is the preimage of $n = 4$?

4. Give the prime decomposition of $\prod_{i=3}^7 (2i)$.

5. Solve the congruence $5x = 1 \pmod{12}$.

6. Compute the following sum $\sum_{i=2}^5 (2i)^2$.

• Write down your **solutions** (+2pt) **and justify** (+1pt).

1. Give a big-O estimate for $f(n) = (n^2 + n \log n)(3n - 100)$.

2. Prove that if n is an integer, then 3 divides $n^3 - n$.

3. Let $0 < d < n$ and k be integers.

How many positive integers not exceeding kn are congruent to d ?