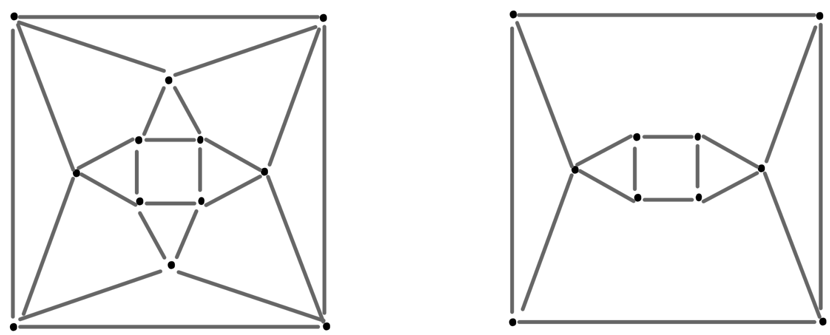
MATH 363 Discrete Mathematics
Assignment 11

Due by April 6th

1. Consider the following two graphs:



   i) (**2pt** each) Find a Hamiltonian path in each graph.

   ii) (**2pt** each) Find an Eulerian path or cycle in each graph. If it is not possible, explain why.

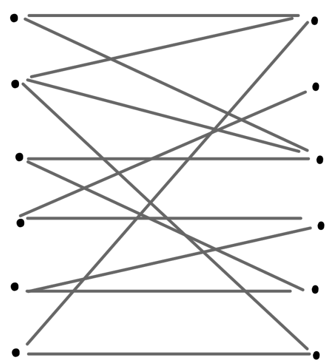
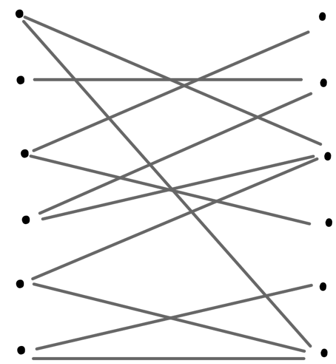
2. (**3pt**) Prove that if the minimum degree of a graph G is at least 2, then G has a cycle.

3. (**2pt** each) Draw two of the 5 platonic solids as planar graphs, verify euler's formula for those graphs and color them with 4 colors.

4. (**2pt**) Show that the n-cube is 2-colourable.

5. (**2pt** each) Select two of the following applications to graph colorings; explain what is the problem and how it can be modeled and solved using graphs, give an example: Exam scheduling, Radio frequency assignments, index registers, solving sudoku puzzles.

6. (**2pt** each) Given a graph $G = (V, E)$, a *complete matching* $M \subset E$ is a subset of the edges in G such that every vertex in V is incident to exactly one of the edges in M.
   Find a complete matching for the following two graphs; if it is not possible, use Hall's theorem to prove it.

7. Suppose there are n people in a group, each aware of a scandal no one else in the group knows about. These people communicate by telephone; when two people in the group talk, they share information about all scandals each knows about. For example, in the first call, each of these people knows about two scandals.

   i) (**1pt**) How many calls are used if we simply have every person call one person, a 'busy body', and then have that person call everyone back?

   ii) (**1pt**) Represent the calls with a graph where edges are numbered in the order the calls are placed.

8. The gossip problem asks for $G(n)$, the minimum number of telephone calls that are needed for all n people to learn about all the scandals.

   i) (**1pt**) Compute $G(1), G(2)$ and $G(3)$; draw graphs with numbered edges to represent your solutions.

   ii) (**1pt**) Does the 'busy body' model above attains the minimum number of calls for $n = 4$. If not, give a better model.

   iii) (**2pt**) Prove by induction that $G(n) \leq 2n - 4$ for $n \geq 4$.
   Hint: In the inductive step, have a new person call a particular person at the start and at the end.

   iv) (**2pt**) Draw the graphs representing the inductive step above for $n = 5, 6$.