

MATH 363 Discrete Mathematics

Assignment 2

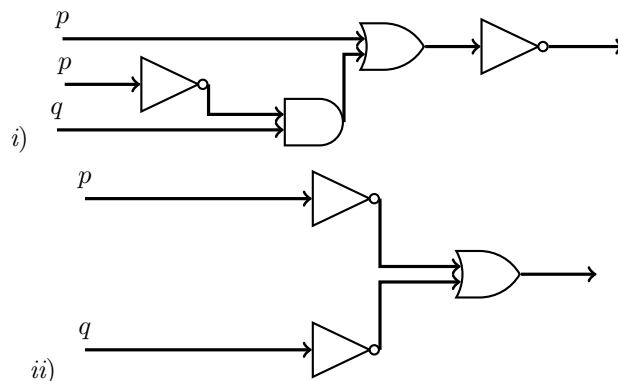
Due by January 27th

1. **(2pt each)** Express each of these specifications using propositional logic (Hint: define 3 propositions and write the statements below in terms of the propositions you define.)

- i)* If the system is in multiuser state, the kernel is functioning.
- ii)* The kernel is not functioning or the system is in interrupt mode.
- iii)* If the system is not in multiuser state, then it is in interrupt mode.
- iv)* The system is not in interrupt mode.

\rightsquigarrow **Definition:** A collection of (compound) propositions is *consistent* if there is an assignment of truth values to each of the proposition variables involved such that every proposition in the collection is true.

2. **(3pt)** Determine whether the specifications in the previous exercise is a consistent collection of propositions.
3. **(3pt each)** Find out the output of these logic circuits.



4. **(3pt each)** Construct a logic circuit using inverters, OR gates, and AND gates that produces the output $(p \wedge \neg r) \vee (\neg q \wedge r)$ from input bits p, q , and r .

5. **(3pt each)** Negate the statements, and find a counterexample to either the statement or its negation.

- i)* $\forall x \in \mathbb{R} (|x| > 0)$
- ii)* $\forall \text{ integer } x > 4, (x^2 \leq 10)$
- iii)* $\exists x \in \mathbb{R} \forall y \in \mathbb{R} (xy = 1)$

6. **(4pt each)** Express each of these system specifications using predicates, quantifiers and logical connectives.

- i)* No directories in the file system can be opened and no files can be closed when system errors have been detected.
- ii)* The file system cannot be backed up if there is a user currently logged on.

7. **(2pt)** Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall xP(x) \leftrightarrow \forall xQ(x)$ are logically equivalent. Justify your answer.