MATH 363 Discrete Mathematics
Assignment 7

Due by March 9th

1. (**2pt**) Let $\alpha = \frac{1+\sqrt{5}}{2}$. Recall that the first Fibonacci numbers are $f_0 = 0$, $f_1 = 1$, $f_2 = 1$. Use induction to prove that for any integer $n \geq 3$, f_n satisfies

$$f_n > \alpha^{n-2}$$

2. (**3pt each**) Give the pseudocode of a recursive algorithm to:

   - compute the greatest common divisor of two distinct positive integers a, b.
   - give a factorization into prime factors of a positive integer n.

3. (**3pt**) Let z_n denote the number of zero-one strings of length n which do not contain two consecutive zeros. Find a recursive formula for z_n and justify this formula.

4. (**2pt** each) Let the functions f, g satisfy the following recursive relation:

   - $f(n) = 9f(n/3) + 20n^2$ when $n = 3k$,
   - $g(n) = 16g(n/4) + n$ $n = 4k$.

   Give a big-O estimate for f and g.