

MATH 363 Discrete Mathematics

Assignment 8

Due by March 16th

1. **(2pt)** How many bit strings of length 10 either start with the bits 101 or end with the bits 111?
2. **(3pt)** How many bit strings of length 10 contains 5 consecutive zeros?
3. **(2pt)** How many bit strings of length 10 contains either 5 consecutive zeros or 5 consecutive ones?
4. **(2pt)** A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?
5. **(2pt each)** Prove $2^n = \sum_{j=0}^n \binom{n}{j}$
 - i) Using the binomial theorem
 - ii) Using a combinatorial proof
6. **(2pt)** Let $n, m \geq k \geq 1$ be integers. Using a combinatorial proof, prove that

$$\binom{m+n}{k} = \sum_{j=0}^k \binom{m}{j} \binom{n}{k-j}.$$

Hint: Pick a committee of a given size, out of a group of men and women.

7. **(2pt)** Let $n, k \geq 1$ be integers. Using a combinatorial proof, prove that

$$\binom{n+k+1}{k} = \sum_{j=0}^k \binom{n+j}{j}.$$

Hint: Consider bit strings with a fixed number of zeros. Then divide in cases according to the place of the last bit with a one.

8. **(2pt each)** Let $m, n \geq 1$ be integers. Consider paths in a grid connecting the points $(0, 0)$ and (m, n) made out of steps either one unit to the right or one unit upward (No moves to the left or downward are allowed).
 - i) Show that there are $\binom{n+m}{n}$ distinct such paths.
(Hint: Represent each path as a bit string encoding the steps going either up or to the right.)
 - ii) Use this experiment to prove Pascal's identity.
(Hint: Consider that each path either passes through $(0,1)$ or $(1,0)$.)
9. **(3pt)** Maria will read a book with 16 chapters during her free time over 5 weekends, she will read in order and at least one chapter per week. In how many ways can she read the book? (e.g. Week 1 Chapters 1-4, Week 2 Chapters 5-10, Week 3 Chapter 11, Week 4 Chapters 12-15, Week 5 Chapter 16)
10. **(2pt each)** A coin is flipped 10 times (it lands either tails or heads with equal probability)
 - i) How many possible outcomes are there in total?
 - ii) What is the probability that exactly 3 heads comes up?
 - iii) What is the probability that the outcomes give either exactly 3 heads or exactly 4 tails?