MATH 363 Discrete Mathematics
SOLUTIONS: Assingment 1

1 Grading scheme

1. Full marks if IS/IS NOT A PROPOPOSITION is correct. Leave a note if the justification is incorrect.
2. Full marks if it is a proposition equivalent to those below. Leave a note if the language is ambiguous.
3. Full marks if answer is as below.
   -1pt if there are parenthesis missing (except in question 3.i))
4. 2pt if stated TRUE.
   1pt if explained why it is TRUE.
5. 3pt if Truth tables are correct.
   1pt if it is stated that ARE/ARE NOT EQUIVALENT.
   -1pt for every pair of typos in each truth table.
6. Full marks if it is stated that IS /IS NOT TAUTOLOGY.
   Leave a note if there is no reference to which assignment of truth values make the proposition false.
   -1pt for every pair of typos in each truth table.

Extra points +2pt If the sequence of logical equivalences is correct.
+2pt If it is stated which logical equivalence law was used at each step.
-1pt If more than 2 of the stated logical equivalence law are incorrect/incomplete.
* Stating the use of Double negation law is not mandatory.

2 Assignment with solutions

1. (**1pt each**) Which of these are propositions? What are the truth values of those that are propositions?

   i) What time is it?
   Not a proposition: it does not state a fact

   ii) The moon is made of green cheese.
   A false proposition

   iii) $2^n \geq 100$.
   Not a proposition: The fact is neither true or false

   Consider the following propositions
   p : I bought a lottery ticket this week.
   q : I won the million dollar jackpot on Friday.
   r : You get an A on the final exam.
   s : You do every exercise in the textbook.
   t : You get an A in this class.

2. (**2pt each**) Express each of these propositions as an English sentence.

i) $\neg p \wedge \neg q$
I didn't buy a lottery ticket this week and I didn't win the million dollar jackpot on Friday.

ii) $\neg p \vee (p \wedge q)$
Either I bought the lottery ticker and won the million dollar jackpot, or I didn't buy the lottery ticket.

iii) $\neg(p \vee q)$
It is not the case that either: I bought the lottery ticket or I won the million dollar jackpot on Friday
Also: same answer as in *i)*

3. (**2pt each**) Write these propositions using r, s, t and logical connectives.

   i) You get an A on the final, you do every exercise in the textbook but you don't get an A in this class.
   $r \wedge s \wedge \neg t$

   ii) Getting an A on the final and doing every exercise in the textbook is sufficient for getting an A in this class.
   $(r \wedge s) \rightarrow t$

   iii) You will get an A in this class if and only if you either do every exercise in the textbook or you get an A on the final.
   $t \leftrightarrow (r \vee s)$

4. (**3pt each**) Determine whether these biconditionals are true or false.

   i) $2 + 2 = 4$ if and only if $1 + 1 = 2$
   True: both propositions are true.

   ii) $0 > 1$ if and only if $2 > 3$
   True: both propositions are false.

   iii) $2 = 5$ if and only if $8 - 3 = 4$
   True: both propositions are false.

5. (**4pt each**) Determine whether the following compound propositions are logically equivalent.

   i) $p \rightarrow \neg p$ and $(p \vee q) \rightarrow (p \wedge q)$
   They are not logically equivalent. See Truth Table 1

   ii) $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$
   They are logically equivalent. See Truth Table 2

6. (**4pt each**) Determine whether the following compound propositions are tautologies.
   (**+4pt** if you use logical equivalences instead of truth tables)

   i) $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
   This is not a tautology. See Truth Table 3

   ii) $[p \wedge (p \rightarrow q)] \rightarrow q$
   This is a tautology. See Truth Table 4
   Alternative answers using logical equivalences in next section

Tables

1.

p	q	$\neg p$	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$	$p \rightarrow \neg p$
T	T	F	T	T	T	F
T	F	F	T	F	F	F
F	T	T	T	F	F	T
F	F	T	F	F	T	T

2.

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

3.

p	q	$\neg p$	$p \to q$	$\neg p \wedge (p \to q)$	$\neg q$	$[\neg p \wedge (p \to q)] \to \neg q$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

4.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Extra Points

6. *i)* $(\neg p \wedge (p \to q)) \to \neg q \equiv p \vee \neg q$. The latter is not a tautology.

Proof.

$$
\begin{aligned}
& (\neg p \wedge (p \to q)) \to \neg q && \text{by Implication law}\\
\equiv\ & [\neg p \wedge (\neg p \vee q)] \to \neg q && \text{by Implication law}\\
\equiv\ & \neg[\neg p \wedge (\neg p \vee q)] \vee \neg q && \text{by 1st De Morgan's law}\\
\equiv\ & [p \vee \neg(\neg p \vee q)] \vee \neg q && \text{by 2nd De Morgan's law}\\
\equiv\ & [p \vee (p \wedge \neg q)] \vee \neg q && \text{by Commut. + Assoc. laws}\\
\equiv\ & (p \vee \neg q) \vee (p \wedge \neg q) && \text{by 1st Distributive law}\\
\equiv\ & [(p \vee \neg q) \vee p] \wedge [(p \vee \neg q) \vee \neg q] && \text{by Commut. + Assoc. laws}\\
\equiv\ & [(p \vee p) \vee \neg q] \wedge [p \vee (\neg q \vee \neg q)] && \text{by Idempotent law}\\
\equiv\ & [p \vee \neg q] \wedge [p \vee \neg q] && \text{by Idempotent law}\\
\equiv\ & p \vee \neg q
\end{aligned}
$$

□

6. *ii)* $(p \wedge (p \to q)) \to q \equiv \mathbf{T}$; which is a tautology.

Proof 1.

$$
\begin{aligned}
& [p \wedge (p \to q)] \to q && \text{by Implication law}\\
\equiv\ & [p \wedge (\neg p \vee q)] \to q && \text{by 2nd Distributive law}\\
\equiv\ & [(p \wedge \neg p) \vee (p \wedge q)] \to q && \text{by Negation law}\\
\equiv\ & [\mathbf{F} \vee (p \wedge q)] \to q && \text{by Domination law}\\
\equiv\ & (p \wedge q) \to q && \text{by Implication law}\\
\equiv\ & \neg(p \wedge q) \vee q && \text{by 1st De Morgan law}\\
\equiv\ & (\neg p \vee \neg q) \vee q && \text{by Associative law}\\
\equiv\ & \neg p \vee (\neg q \vee q) && \text{by Negation law}\\
\equiv\ & \neg p \vee \mathbf{T} && \text{by Domination law}\\
\equiv\ & \mathbf{T}
\end{aligned}
$$

$\square$

Proof 2.

$$
\begin{aligned}
& [p \wedge (p \to q)] \to q && \text{by Implication law} \\
\equiv\ & \neg[p \wedge (p \to q)] \vee q && \text{by 1st De Morgan's law} \\
\equiv\ & [\neg p \vee \neg(p \to q)] \vee q && \text{by Commut. + Assoc. laws} \\
\equiv\ & [\neg p \vee q] \vee [\neg(p \to q) \vee q] && \text{by Implication law} \\
\equiv\ & [\neg p \vee q] \vee [\neg(\neg p \vee q) \vee q] && \text{by 2nd De Morgan's law} \\
\equiv\ & [\neg p \vee q] \vee [(p \wedge \neg q) \vee q] && \text{by 1st Distributive law} \\
\equiv\ & [\neg p \vee q] \vee [(p \vee q) \wedge (\neg q \vee q)] && \text{by Negation law} \\
\equiv\ & [\neg p \vee q] \vee [(p \vee q) \wedge \mathbf{T}] && \text{by Domination law} \\
\equiv\ & [\neg p \vee q] \vee [p \vee q] && \text{by Commut. + Assoc. laws} \\
\equiv\ & [\neg p \vee p] \vee [q \vee q] && \text{by Negation law} \\
\equiv\ & \mathbf{T} \vee [q \vee q] && \text{by Idempotent law} \\
\equiv\ & \mathbf{T} \vee q && \text{by Domination law} \\
\equiv\ & \mathbf{T}
\end{aligned}
$$

$\square$