Relations-Graphs-Trees review

1. Definitions
2. Understand theorems/algorithms
3. Apply theorems/algorithms

Relation definitions

- Relation from set A to set B, and relation on A,
- Properties:

  Reflexive, transitive, symmetric, antisymmetric.

  Equivalence relation.
- The digraph and matrix representation of a relation.

Graphs defintions (1)

For graphs with no loops nor multiple edges

- Connected components of a graph
  - The partition of vertices into connected components
  - The neighbourhood of a vertex
  - Degree of a vertex
  - Isolated vertices, $\deg(v) = 0$
- K_n, C_n, H_n and $K_{n,m}$ (complete bipartite)
- Bipartite graphs

Graphs defintions (2)

- Paths (closed walk, cycle, simple path, trail).
- Eulerian and Hamiltonian path /circuits
- A complete Matching
- Planar graphs
  - A face in a planar graph.
  - The outer face of a planar graph.
- A proper colouring of
  - the vertices in a graph / faces in a planar graph

Tree definitions

- A tree:
  rooted tree, m-ary tree, full m-ary tree and forest.
- Vertices:
  leaf (external), internal vertices,
  parent and children of a vertex.
- The depth of a vertex; height of a rooted tree.
- A spanning tree of a connected graph.
- A minimum spanning tree of a graph with weighted edges.

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- Represent problems with relations; e.g. student/class, city/state, etc.

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- Use that relation R^n connects the endpoints of paths of length n in the digraph corresponding to R.

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- 4-colour thm: coloring faces of planar graphs.

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  - Kruskal's algorithm.

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- Given a problem involving graphs or trees,
  determine which of the algorithms/theorems above can be applied.