Name: **Recitation Section:** J

Math 1553 J1-J3 Quiz : Sections 1.7-1.9

Solutions

The quiz has a total of 12 points and you have 10 minutes. Read carefully and Clearly justify how you obtained your answers.

1. [2 points each] Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be defines as $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+3y-z \\ 4x+6y-2z \end{pmatrix}$

a) Find the standard matrix for the linear one.

b) Draw a picture of the range of the linear one.

c) Is the linear one onto? If so, why? If not, find a vector b in $\mathbf{R}^2$ which is not in the range. (It is enough to use the picture in (b).)

Solution.

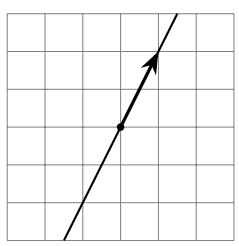
a) We have to plug in the unit coordinate vectors to get the columns:

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}.$$

Therefore the standard matrix for T is

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 6 & -2 \end{pmatrix}.$$

b) The range of T is the span of the columns of the standard matrix. All three columns lie on the line spanned by $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, so the range is just this line.



c) The range of T is a line in $\mathbf{R}^2$, so it is strictly smaller than the codomain. Hence T is not onto. Looking at the picture, we see that, for instance, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is not in the range.

2. [2 points each] In each of the following, provide a transformation with the required property.

a) Give an example of $U : \mathbf{R}^2 \to R^2$ that is not linear.

b) Given an example of $T : \mathbf{R}^3 \to R^2$ that linear and is onto.

c) For your example in (b), is $\{T(e_1), T(e_2), T(e_3)\}$ linearly independent? If not, provide a linear combination that equals the zero vector.

Solution.

a) Let $U\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} x_1 + 1 \\ x_2^2 \end{pmatrix}$. One property of linear trasnformations is that the zero vector is mapped to the zero vector. Since $U\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then U is not linear.

b) Let $T(x) = Ax$ with matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. Since T is defined by matrix multiplication, it is linear. T is onto because the columns of A span $\mathbf{R}^2$.

c) The following is a linear combination of $\{T(e_1), T(e_2), T(e_3)\}$ that equals zero:

$$1 \cdot T(e_1) + 1 \cdot T(e_2) - 1 \cdot T(e_3) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$$

Therefore, the set is not linearly independent.