Name: **Recitation Section:** J

Math 1553 J1-J3 Quiz : Sections 5.1-5.2

Solutions

The quiz has a total of 10 points and you have 10 minutes. Read carefully.

1. [2 points each] Justify your work, you can state any theorem or statement from the lecture notes.

a) Is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ in the 1-eigenspace of the matrix $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$?

b) Write down the definition of characteristic polynomial of matrix B.

Solution.

a) Yes. $A\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, so $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector with eigenvalue 1.

2 pt. if answer is yes and there is a reason (e.g. matrix multiplication)
1 pt. if reason/work is missing

b) The characteristic polynomial of B is the function $f(\lambda) = \det(B - \lambda I)$.

up to 1 pt. if they attempt any approach
2pt for correct formula

2. [3 points each] Justify your work, you can state any theorem or statement from the lecture notes.

a) If v_0 is an eigenvector of B with eigenvalue 3, compute $v_4 = B^4 v_0$

b) Find a 2×2 matrix whose characteristic polynomial is

$$f(\lambda) = \lambda^2 + 9$$

Solution.

a) We can repetively apply the identity $Bv_0 = 3v_0$. Then $v_4 = 81v_0$ since

$$B^4 v_0 = B^3(Bv_0) = 3B^3 v_0 = 3B^2(Bv_0) = 9B^2 v_0 = 27Bv_0 = 81v_0$$

2 pt. if they mention or write formulas for difference equations
3 pt if the answer does not include any power of matrix B
-1 pt if there is no justification for the answer

b) It is not difficult to guess the entries of the matrix. Or you can use the formula for 2×2 matrices $f(\lambda) = \lambda^2 - \mathrm{tr}(A)\lambda + \det(A)$ to devise a matrix whose diagonal entries sum zero and the determinant is 9. Two examples are $A = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix}$ and $A = \begin{pmatrix} 0 & -1 \\ 9 & 0 \end{pmatrix}$.

up to 2 pt. if they attempt any approach
3 pt if the matrix has the correct characteristic polynomial