## Math 1553 Worksheet §2.1, 2.2, 2.3

1. If $A$ is a $3 \times 5$ matrix and $B$ is a $3 \times 2$ matrix, which of the following are defined?
a) $A-B$
b) $A B$
c) $A^{T} B$
d) $B^{T} A$
e) $A^{2}$
2. Find all matrices $B$ that satisfy

$$
\left(\begin{array}{cc}
1 & -3 \\
-3 & 5
\end{array}\right) B=\left(\begin{array}{cc}
-3 & -11 \\
1 & 17
\end{array}\right)
$$

3. a) If the columns of an $n \times n$ matrix $Z$ are linearly independent, is $Z$ necessarily invertible? Justify your answer.
b) Solve $A B=B C$ for $A$, assuming $A, B, C$ are $n \times n$ matrices and $B$ is invertible. Be careful!
4. True or false (justify your answer). Answer true if the statement is always true. Otherwise, answer false.
a) If $A$ is an $m \times n$ matrix and $B$ is an $n \times p$ matrix, then each column of $A B$ is a linear combination of the columns of $A$.
b) If $A$ and $B$ are $n \times n$ and both are invertible, then the inverse of $A B$ is $A^{-1} B^{-1}$.
c) If $A^{T}$ is not invertible, then $A$ is not invertible.
d) If $A$ is an $n \times n$ matrix and the equation $A x=b$ has at least one solution for each $b$ in $\mathbf{R}^{n}$, then the solution is unique for each $b$ in $\mathbf{R}^{n}$.
e) If $A$ and $B$ are invertible $n \times n$ matrices, then $A+B$ is invertible and $(A+B)^{-1}=$ $A^{-1}+B^{-1}$.
f) If $A$ and $B$ are $n \times n$ matrices and $A B x=0$ has a unique solution, then $A x=0$ has a unique solution.
5. Suppose $A$ is an invertible $3 \times 3$ matrix and

$$
A^{-1} e_{1}=\left(\begin{array}{l}
4 \\
1 \\
0
\end{array}\right), \quad A^{-1} e_{2}=\left(\begin{array}{l}
3 \\
2 \\
0
\end{array}\right), \quad A^{-1} e_{3}=\left(\begin{array}{l}
0 \\
0 \\
1
\end{array}\right)
$$

Find $A$.

