# Math 1553 Worksheet §5.1, 5.2

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. In every case, assume that $A$ is an $n \times n$ matrix.

   a) If $v_1$ and $v_2$ are linearly independent eigenvectors of $A$, then they must correspond to different eigenvalues.

   b) The entries on the main diagonal of $A$ are the eigenvalues of $A$.

   c) The number $\lambda$ is an eigenvalue of $A$ if and only if there is a nonzero solution to the equation $(A - \lambda I)x = 0$.

   d) To find the eigenvectors of $A$, we reduce the matrix $A$ to row echelon form.

   e) If $A$ is invertible and 2 is an eigenvalue of $A$, then $\dfrac{1}{2}$ is an eigenvalue of $A^{-1}$.

2. In what follows, $T$ is a linear transformation with matrix $A$. Find the eigenvectors and eigenvalues of $A$ without doing any matrix calculations. (Draw a picture!)

   a) $T$ = identity transformation of $\mathbf{R}^3$.

   b) $T$ = projection onto the $xz$-plane in $\mathbf{R}^3$.

   c) $T$ = reflection over $y = 2x$ in $\mathbf{R}^2$.

3. Let $A = \begin{pmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{pmatrix}$. Find the eigenvalues of $A$.

## Supplemental Problems

For those who want additional practice problems after completing the worksheet, here are some extra practice problems.

**1.** Find a basis $\mathcal{B}$ for the $(-1)$-eigenspace of $Z = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$

**2.** Suppose $A$ is an $n \times n$ matrix satisfying $A^2 = 0$. Find all eigenvalues of $A$. Justify your answer.

**3.** Give an example of matrices $A$ and $B$ which have the same eigenvalues and the same algebraic multiplicities for each eigenvalue, but which are *not* similar. Justify why they are not similar.

**4.** Using facts about determinants, justify the following fact: if $A$ is an $n \times n$ matrix, then $A$ and $A^T$ have the same characteristic polynomial.