Math 1553 Worksheet §5.3, 5.5

1. Answer yes / no / maybe. In each case, A is a matrix whose entries are real.

a) If A is a 3×3 matrix with characteristic polynomial $-\lambda(\lambda-5)^2$, then the 5-eigenspace is 2-dimensional.

b) If A is an invertible 2×2 matrix, then A is diagonalizable.

c) Can a 3×3 matrix A have a non-real complex eigenvalue with multiplicity 2?

d) Can a 3×3 matrix A have eigenvalues $3, 5$, and $2+i$?

2. Let $A = \begin{pmatrix} 8 & 36 & 62 \\ -6 & -34 & -62 \\ 3 & 18 & 33 \end{pmatrix}$.

The characteristic polynomial for A is $-\lambda^3 + 7\lambda^2 - 16\lambda + 12$, and $\lambda - 3$ is a factor. Decide if A is diagonalizable. If it is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

3. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.

a) Find all eigenvalues and eigenvectors of A.

b) Write $A = PCP^{-1}$, where C is a rotation followed by a scale. Describe what A does geometrically. Draw a picture.

Supplemental Problems

For those who want additional practice problems after completing the worksheet, here are some extra practice problems.

1. Let A and B be 3×3 real matrices. Answer yes / no / maybe:

a) If A and B have the same eigenvalues, then A is similar to B.

b) If A and B both have eigenvalues $-1, 0, 1$, then A is similar to B.

c) If A is diagonalizable and invertible, then A^{-1} is diagonalizable.

2. Give an example of a non-diagonal 2×2 matrix which is diagonalizable but not invertible. Justify your answer.

3. Suppose A is a 7×7 matrix with four distinct eigenvalues. One eigenspace has dimension 2, while another eigenspace has dimension 3. Is it possible that A is not diagonalizable?

4. Let $A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$.

a) Find all (complex) eigenvalues and eigenvectors of A.

b) Write $A = PCP^{-1}$, where C is a block diagonal matrix, as in the slides near the end of section 5.5.

c) What does A do geometrically? Draw a picture.